Baryons with two heavy quarks

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Abstract

We consider general physical characteristics of doubly heavy baryons: the spectroscopy in the framework of potential approach and sum rules of QCD, mechanisms of production in various interactions on the basis of fragmentation model with account of preasymptotic corrections caused by higher twists over the transverse momentum of baryon, inclusive decays and lifetimes in the operator product expansion over the inverse powers of heavy quark masses as well as the exclusive decays in the sum rules of QCD. We generalize the methods developed in the effective theory of heavy quarks towards the description of systems with two heavy quarks and a single light quark. The calculations are presented for the masses, decay widths and yields of baryons with two heavy quarks in the running and planned experimental facilities. We discuss the prospects of search for the baryons and possibilities of experimental observation. The most bright physical effects concerning these baryons as well as their position in the system for the theoretical description of heavy quark dynamics are considered.

Contents

Introduction.

After the precision investigations of neutral intermediate Z boson at LEP (CERN) and observation of t quark at FNAL were done, the study of electroweak interactions in the sector of heavy quarks is a task among the most actual problems in the physics of fundamental particles, since in the framework of standard model [1] such the searches can result in a complete picture of effects responsible for the unreversability of time at energies below the scale of electroweak symmetry breaking. A comparative analysis of processes with violation of combined CP parity inverting both the charges (C) and space orientation (P) in decays of hadrons containing the heavy quarks becomes accessible for quite precise measurements due to putting into action specialized facilities Belle (KEK) and BaBar (SLAC) along with the upgraded detectors of CDF and D0 at FNAL. The results of such experiments probably allow us to add an essential missing chain-cell in the standard model of interactions, i.e. a complete description of charged currents for three quark generations [2]. This study is the most important problem along with both the observation of scalar Higgs particles providing the mechanism of electroweak symmetry breaking, and the investigation of neutrino currents.

The pricision study of electroweak properties of heavy quarks make rise a deep theoretical problem, that is the description of dynamics for the strong interactions of quarks, since the strong forces cause the formation of bound states, hadrons: mesons and baryons, and the observable quantities like characteristics of rare decays with the CP violation are assigned to the bound states, so that we need definite and reliable representations about the relation of measured properties with the parameters of heavy quark interactions themselves. In this way, fine effects of electroweak physics can be extracted under a high precision description for the dominant contributions of quantum chromodynamics (QCD). In this respect, we deal with a general problem on quantitative understanding the quark confinement in QCD, that can be efficiently investigated not only in the spectroscopy and processes with the production and decays of exotic hybrid and glueball states [3], but also in the study of hadrons with heavy quarks. In practice, the measured quantities of asymmetries in the decays of heavy hadrons, for instance, are expressed in termes of functions parametrically depending on both the primary characteristics assigned to charged and flavor changing neutral currents of quarks, and the hadronic matrix elements of quark operators, which usually cannot be straightforwardly determined from a wide set of various experimental data. So, we need a sound theoretical anlaysis of such the matrix elements in QCD.

A more complete list of heavy quark bound states under study provides a more wide region for the variation of conditions, wherein the forces of QCD act on the heavy quarks, so that the theoretical methods for the description of hadrons containing the heavy quarks should be more accomplished in order to get a consistent understanding of various quark systems. In this way, we see a new field of activity is baryons, containing two heavy quarks. So, the theoretical predictions for the properties of such the baryons are of interest and actual. The doubly heavy baryons naturally continue the list of long-lived heavy hadrons with both a single heavy quark (D, B mesons and Λ_c, Σ_c, Ξ_c, Ω_c, Λ_b baryons) and two heavy quarks (the B_c meson). With respect to the character of strong interactions, these baryons could have common features with the heavy quarkonia $\bar{c}c$ and $\bar{b}b$. In practice, we could expect experimental observation of doubly heavy baryons in the searches at the modern hadron colliders with high lumonosities (Tevatron, LHC), since the yield of such baryons should be of the same order of magnitude as for the doubly heavy meson containing the quarks of different flavors, the B_c meson[1], while the methods for the registration of rare decays with heavy particles recently get a high efficiency due to a development in the technique of vertex detectors as

[1]See the review on the physivs of B_c in [4].

was successfully exhibited in the first experimental observation of B_c by the CDF collaboration [5].

Constructing the theoretical methods for the description of QCD dynamics with the heavy quarks is based on a clear physical definition: the quark Q is ascribed heavy, if its mass m_Q is much greater than the scale of heavy quark confinement in the bound state, so $m_Q \gg \Lambda_{QCD}$. Thus, considering the problem of strong interactions with the heavy quarks, i.e. calculating hadronic matrix elements of quark operators, we could involve a small parameter $\Lambda_{QCD}/m_Q \ll 1$, which could serve for the development of formal approximate methods.

So, first of all, in hard processes with virtualities of the order of heavy quark masses, say, in the production of heavy quarks, the QCD coupling constant is small, $\alpha_s \sim \frac{1}{\ln m_Q/\Lambda_{QCD}}$, and the usual technique of perturbative theory in powers of α_s is justified. Another productive method is the operator product expansion (OPE) in inverse powers of heavy quark mass. In such the approach a calculation of hadronic matrix element for a quark operator leads to summing up the matrix elements of operators, whose properties assume a hierarchy in terms of small parameter $\Lambda_{QCD}/m_Q \ll 1$, i.e. a suppression of some contributions by powers of Λ_{QCD}/m_Q, since the interaction in the bound state containing the heavy quark is characterized by the energies close to Λ_{QCD} determining the inverse size of the hadron. This expansion gives a complete description of hadronic systems containing a single heavy quark. If there are two heavy quarks in the hadron, then along with the scale of nonperturbative interactions another energetic scale is a momentum transfer in the coulomb interaction with a virtuality of $\mu \sim \alpha_s m_Q$, so that a relative velocity for two heavy quarks moving inside the hadron, v, is determined by a comparatively small value of coupling constant in QCD, and $v \sim \alpha_s$, where the coupling constant is taken at the scale of virtualities prescribed to the coulomb interactions. Thus, in the heavy quarkonium $\bar{Q}Q'$, say, there is an additional small parameter, that is the relative velocity of nonrelativistic quarks v, which can be used in the OPE in order to calculate the hadronic matrix elements. Following such the approach, we underline three methods for the evaluation of quantities characteristic for the bound states with heavy quarks:

- the *operator product expansion* in the inverse powers of heavy quark mass in QCD for calculations of inclusive width and lifetimes of heavy hadrons, where terms correcting the leading approximation are given by some external parameters [6];

- *sum rules* of QCD and nonrelativistic QCD (NRQCD) for both two-point correlators of quark currents in spectroscopic calculations and three-point correlators in estimations of exclusive decay modes [7];

- *potential models* used for the evaluation of exclusive characteristics of hadrons containing the heavy quarks [8].

Let us stress, first, that the sum rules are based on the operator product expansion, too. However, the only external parameters of sum rules are fundamental quantities such as the masses of heavy quarks, the normalization of QCD coupling constant and the quark-gluon condensates in contrast to inclusive estimates in the general OPE, wherein we should use, for instance, the value of heavy quark binding energy in the hadron, the average square of heavy quark momentum etc. Second, in the expansion over the inverse powers of heavy quark mass as well as over the relative velocity of heavy quarks inside the hadron an important role is played by both the perturbation theory and the renormalization group relations, which are necessary for the calculation of Wilson coefficients in the operator product expansion, since these coefficients enter the factors in front of operators or matrix elements under consideration.

It is worth to emphasize that in the method of OPE for the heavy quarks we could consider an actual operator, say, a current of weak decay or a product of currents as it does in sum rules of QCD, with the consequent expansion, while the approach of effective field theory could be useful, too, wherein the starting point of construction is a formal expansion of QCD lagrangian itself for the heavy quarks. Then we can isolate a leading term in the effective lagrangian and treat the rest of terms as perturbations in the expansion. In this way the definition of leading term is determined by the character of the problem, i.e. by the actual convergency in the estimation of physical quantities calculated in the effective lagrangian. So, for the hadrons with a single heavy quark the effective theory of heavy quarks (HQET) [9] was developed. In this theory we can neglect the binding energy of heavy quark inside the hadron to the leading order, therefore, the kinetic energy of heavy quark is the perturbation. It is important to note that, first, in the leading order of HQET the effective lagragian possesses the following symmetries: *a*) the heavy quarks with identical velocities equal to the velocity of hadron, where the quarks are bound, can be permuted, that implies the heavy flavor symmetry, b) the heavy quark spin is decoupled from the interaction with low-virtual gluons, since the current is given by the quark velocity v_ν, that implies the spin symmetry. Second, the leading term of effective lagrangian provides the renormalization group behaviour different from the full QCD. Particularly, the currents being conserved in full QCD and, hence, having zero anomalous dimensions become divergent after the transition to the fields of effective theory. The same note concerns to the correction terms of effective lagrangian: the corresponding Wilson coefficients in the effective theory have nonzero anomalous dimensions, too. Thus, we deal with the situation, wherein we need an infinite number of normalization conditions for the anomalous Wilson coefficients in the renormalization group in order to correctly define the theory. This problem has a clear physical reason, since in the effective theory constructed for the fields with low virtualities one should introduce a cut off in the ultraviolet region at the scale of the order of heavy quark mass because at high virtualities the assumptions made in the derivation of the theory are not correct. Since the effective theory was derived from the full QCD, we have to constructively put the effective lagrangian equal to the lagrangian of full QCD at a scale μ_{hard} about the heavy quark mass m_Q. This fact implies that in the given order over the coupling constant of QCD we have to calculate the effective action of QCD with account of corresponding loop corrections and to transform it by the expansion in inverse powers of heavy quark mass, so that the expanded lagrangian should be equalized to the effective lagrangian calculated in the effective theory in the same order of coupling constant with the anomalous Wilson coefficients at the scale μ_{hard}, that results in the matching conditions for the unknown constans of integration in the renormalization group equations for the Wilson coefficients in the effective theory. After the matching is done, we remove an ambiguity in the choice of finite contraterms in the renormalization of effective lagrangian, and it becomes definite at μ below the scale of matching with full QCD μ_{hard}. Thus, for the hadrons containing single heavy quark along with a light one the scheme of effective theory HQET is cosistently constructed.

For the heavy quarkonia composed of heavy quark and heavy anti-quark another physical situation takes place. Indeed, the coulomb interaction of nonrelativistic heavy quarks leads to that the kimetic energy is of the same order of magnitude as the potential energy, while one could naively expect that the kinetic term $\mathbf{p}^2/2m_Q$ should be suppressed by the heavy quark mass. However, since in the coulomb exchange we have $p \sim \alpha_s m_Q$, such the suppression of kinetic energy does not take place, and the wave functions of heavy quarkonia depend on the quark masses, i.e. they are flavor-dependent. In the formal approach of effective theory for the nonrelativistic quarks in the heavy quarkonium the leading term of lagrangian is defined with account of the kinetic energy, so that we deal with the nonrelativistic QCD (NRQCD) [10]. In NRQCD compared with HQET, the

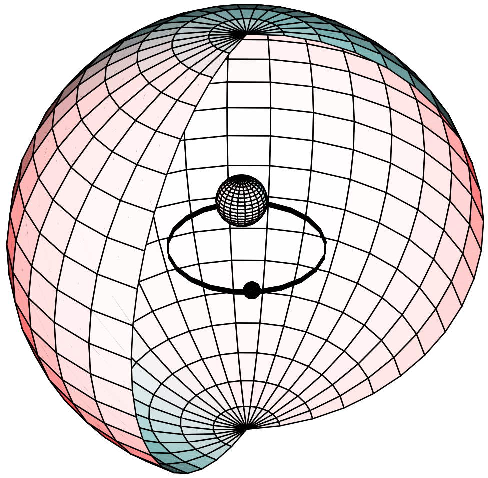

Figure 1. The character of strong interactions in the doubly heavy baryon Ξ_{bc}: the compton lengths of quarks $\lambda_Q = 1/m_Q$, the size of heavy diquark $r_{bc} \sim 1/(v \cdot m_Q)$ and the scale of nonperturbative confinement of light quark $r_{QCD} = 1/\Lambda_{QCD}$ are arranged by $\lambda_b \approx \frac{1}{3}\lambda_c \approx \frac{1}{9} r_{bc} \approx \frac{1}{27} r_{QCD}$.

spin symmetry of leading order effective lagragian survives, while there is no heavy flavor symmetry because the contribution of kinetic energy explicitely depends on the quark masses. In the same manner as in HQET, the Wilson coefficients in the effective lagrangian of NQRCD have to be matched with the full QCD at a scale about the heavy quark mass, so that, in general, these coefficients have anomalous dimensions different from those of HQET, since the kinetic energy results in the ultraviolet behaviour of quark propagators, which is different from the behaviour in HQET.

The potential approach is also based on the operator product expansion. So, the static potential means the expansion of effective action of QCD for two infinitely heavy sources j posed at a fixed distance r, so that the effective action has the form of $\Gamma(j) = -V(r) \cdot T$, where $T \to \infty$ is a time of the sources are switched on. For actual problems the long time interval means that virtualities of external gluon fields interacting with the heavy quarks are much less than the inverse distance, i.e. $\mu \sim \frac{1}{T} \ll \frac{1}{r} \sim m_Q v$. This constraint for the consistency of potential approach can be expressed in terms of effective theory, which is called the potential nonrelativistic QCD (pNRQCD) [11]. The construction of pNRQCD follows the matching of NQRCD action with the effective action of supersoft fields treated in the multipole expansion of QCD with nonrelativistic heavy quarks at $\mu \sim v \cdot m_Q$. The leading order includes the kinetic energy as well as the Wilson coefficient meaning the static potential depending on the distance between the quarks, r. Thus, the potential approach also has the status of OPE, and the static approximation for the potential is determined by the covergency of this expansion in pNRQCD.

In this respect, the baryons with two heavy quarks especially are of interest for the theoretical consideration, since for their description one should develop and use a combined approach involving the features of HQET, NRQCD and pNRQCD because in this systems the interaction of light quark with the heavy quarks is essential as well as the interaction between the heavy quarks (see Fig. 1).

In this review we solve the problem on the description of bound baryonic states $QQ'q = \Xi_{QQ'}$ with two heavy quarks Q, Q' and a light quark q on the basis of factorizing the interactions with various virtualities determined by the following:

- the confinement scale Λ_{QCD} for the nonperturbative interactions of both the heavy quarks with

the light one and the quarks with the quark-lguon condensates,

- the size of heavy diquark $r_{QQ'} \sim 1/(m_{Q,Q'} \cdot v)$, which is composed of two nonrelativistic heavy quarks moving with a small relative velocity $v \ll 1$, for the interactions between the heavy quarks,

- the scale of hard gluon corrections at the energies about the heavy quark masses m_Q,

so that we assume that the leading approximation is valid if there is the hierarchy of QCD interaction scales in $\Xi_{QQ'}$

$$\Lambda_{QCD} \ll m_Q \cdot v \ll m_Q. \tag{1}$$

In this approach the doubly heavy diquark acts as a local heavy source of gluon field for the light quark. This source is charged as the anti-triplet under the color group of QCD, while the diquark itself is the system of two nonrelativistic quarks in the low-frequency field of light quark. Thus, for the motion of light quark and diquark we can use the effective theory of HQET, to the moment for the motion of heavy quarks inside the diquark we should modify NRQCD and pNRQCD in order to consider the nonrelativistic fields of heavy quarks in the anti-triplet state, but the singlet one.

On the basis of quark-diquark representation for the interactions we consider various physical aspects of baryons with two heavy quarks. In **Chapter 1** the mass spectrum of baryons $\Xi_{QQ'}$ is constructed in the potential approach. We calculate the characteristics of groud state and its excitations in the system of heavy diquark as well as in the system of light quark and diquark. We show that there is a family of $\Xi_{QQ'}$ levels with the quasi-stable states for the heavy diquark composed of identical heavy quarks. For these states the Pauli principle dictates quite definite values for the sum of quark spins, so, the total spin equals unit for the P-even wave functions in the configuration space, while it is equal to zero for the P-odd wave functions, since the anti-triplet color state of diquark is anti-symmetric under the permutation of color indices. Taking into account the small size of diquark, the nonrelativistic motion of heavy quarks and a small ratio of Λ_{QCD}/m_Q, the operators for the transitions of excited P-wave diquark into the ground S-wave level with the emission of π meson are suppressed because in this transition both the spin state of diquark and orbital state should change. We determine the region of consistency for the quark-diquark approximation in the calculations of $\Xi_{QQ'}$ mass spectra in the framework of potential approach.

In **Chapter 2** the two-point sum rules of NRQCD are considered for the baryonic currents with two heavy quarks. We discuss a criterium of stability for the results of such the sum rules in estimates of masses and coupling constants of $\Xi_{QQ'}$ baryons. We show that reliable results can be obtained after account of the quark and gluon condensates as well as their product and mixed consdensate, i.e. after the introduction of combined condensates of higher dimensions. We calculate the masses and coupling constants for the ground states of baryons $\Xi_{QQ'}$ and the doubly heavy baryons with strangeness $\Omega_{QQ'}$, too. Reliable predictions for the mass splitting of $M_{\Xi_{QQ'}} - M_{\Omega_{QQ'}}$ are presented. We get anomalous dimensions for the baryonic currents in NQRCD up to two-loop approximation, that allows us to estimate the coupling constants of baryons with the baryonic currents not only in NRQCD but also in full QCD.

Mechanisms for the production of $\Xi_{QQ'}$ baryons are considered in **Chapter 3**. Following the factorization of interactions in the quark-diquark approach, in e^+e^- annihilation at high energies the inclusive production of doubly heavy baryons can be described in the form of subsequent fragmentation for both the heavy quark into the heavy diquark and the diquark into the baryon. In this way the virtualities in the first process are determined by the masses of heavy quarks, so that factorizing the soft movement of heavy quarks inside the diquark we can use the perurbative QCD

and get an analytic form of fragmentation functions for the states with various spins and orbital quantum numbers of diquark. Keeping in mind the difference between the color structures of diquark and heavy quarkonium, these calculations repeat the consideration of fragmentation into the doubly heavy mesons up to a color factor. Supposing the representation of heavy diquark by the local field in its interactions with the light quark and approximating the field components of baryon by its dominant fast valence quarks, we develop a QCD-motivated perturbative model for the fragmentation of diquark into the baryon and calculate an analytic form of fragmentation function for the vector and scalar diquarks entering the baryon $\Xi_{QQ'}$ with the spin $\frac{1}{2}$. The estimates for the pair production of baryons in the e^+e^- annihilation close to the threshold are presented, too. The analysis turns to be more complicated in the consideration of mechanism for the production of baryons $\Xi_{QQ'}$ in hadron collisions due to subprocesses of quark-antiquark annihilation (low energies, fixed target experiments) and gluon-gluon fusion (high energies, hadron colliders). This complication is connected to a greater number of diagrams in the leading approximation, i.e. in the fourth order of QCD coupling constant. Making use of numerical method, we show that at high energies of partonic subprocess and at transverse momenta much greater than the baryon mass the complete set of diagrams in the given order of perturbation theory in QCD leads to the factorization for the production of heavy quark and its fragmentation into the doubly heavy diquark, expressed in the form of univerasl fragmetation function analytically derived in the perturbative QCD. This fact implies the consistency of approach used. The advantage of such considereation with the complete set of diagrams dictated by the gauge invariance is a possibility to calculate not only the leading term with respect to the transverse momentum $p_\perp$, that gives the fragmentation dropping as $\sim \frac{1}{p_\perp^4}$, but also the correction terms, the higher twists over the transverse momentum. In this way we get a definite estimate for the transverse momentum determining the boundary between the regions of fragmentation and recombination (the higher twists). We point out that the statistics of events with the production of doubly heavy baryons $\Xi_{QQ'}$ is dominantly integrated out at small transverse momenta lying in the region of recombination. We represent estimates for the total and differential cross sections of $\Xi_{QQ'}$ baryon production in hadronic experiments at various energies and for the pair production of doubly heavy baryons in the quark-antiquark annihilation.

The method of operator product expansion in the inverse powers of heavy quark masses is exploited in **Chapter 4** for the analysis of lifetimes and inclusive width in decays of $\Xi_{QQ'}$ baryons. In this consideration of mechanisms for the decays of heavy quarks entering the doubly heavy baryons, the following three physical effects are essentially important:

- valuable corrections to the spectator width in decays of heavy quarks appear because of movement of quarks in the heavy diquark, whose size is small and, hence, the relative momenta of quarks $p \sim v \cdot m_Q$ are greater than the momentum of heavy quark in the hadron with the single heavy quark, when $k \sim \Lambda_{QCD}$, since we have $v \cdot m_Q \gg \Lambda_{QCD}$;

- nonspectator contributions due to the Pauli interference between the products of heavy quark decay and the valence quarks in the initial state can give a fraction 30-50% of the total width, so that the feature of anti-symmetric color wave function for the baryons is a possibility of both a positive or negative overall sign for the term with the interference[2];

- along with the Pauli interference the weak scattering of quarks in the initial state appears in the OPE in the form of operator of higher dimension and it is enhanced by a two-particle phase

[2]The overall sign is determined by the anti-symmetric permutation of fermions multiplied by the color factor.

space in comparison with other operators with the same dimension, since they have a three-particle phase space in the final state, so that the three-particle phase space is suppressed in units of heavy quark mass; the weak scattering gives about 30% of total width for the baryons with charmed quark.

In this way, we show how the arrangement of lifetimes is produced for the baryons with two heavy quarks. Then we determine the parametric dependence of estimates for the total and inclusive widths on the physical quantities of hadron system. So, the small size of heavy diquark determines its wave function yielding the factor in the evaluation of nonspectator decays. The masses of heavy quarks in OPE are essentially correlated for the hadrons with different quark contents, so that current experimental data on the semileptonic, nonleptonic and total widths decrease uncertainties of estimates. Furthermore, the experimental data are able to essentially improve the qualitative and quantitative knowledges on the dynamics of heavy hadrons with the single or two heavy quarks.

We analyze exclusive semileptonic decays and nonleptonic decays in the assumption of factorization in the framework of three-point sum rules of NRQCD, which allow us to derive relations for the formfactors of transitions given by hadronic matrix elements from the spin symmetry of effective lagrangian. We discuss uncertainties of calculations and compare the sum rule results with the predictions of potential models for the exclusive decays.

In **Conclusion** we summarize our results on the physics of baryons containing two heavy quarks. The obtained predictions do not only point to a way for a goal-recognized search of such baryons, but also make a basis for a more accomplished and detailed theoretical analysis of physical effects in the hadronic systems with two heavy quarks, which are, no doubts, of interest, particularly, for the reliable predictions of total and exclusive widths. Finally, we consider possibilities for an experimental observation of doubly heavy baryons.

Chapter 1. Spectroscopy of doubly heavy baryons: the potential approach

In this chapter we analyze basic spectroscopic characteristics for the families of doubly heavy baryons $\Xi_{QQ'} = (QQ'q)$, where $q = u,\ d$ and $\Omega_{QQ'} = (QQ's)$.

A general approach of potential models to calculate the masses of baryons containing two heavy quarks was considered in refs. [12]. The physical motivation used the pair interactions between the quarks composing the baryon, that was explored in the three-body problem. Clear implications for the mass spectra of doubly heavy baryons were derived. So, for the given masses of heavy charmed and beauty quarks, the approximation of factorization for the motion of doubly heavy diquark and light quark is not accurate. It results in the ground state mass and excitation levels, essentially deviating from the estimates in the framework of appropriate three-body problem. For example, we can easily find that in the oscillator potential of pair interactions an evident introduction of Jacobi variables leads to the change of vibration energy $\omega \to \sqrt{\frac{3}{2}}\omega$ in comparison with naive expectations of diquark factorization.

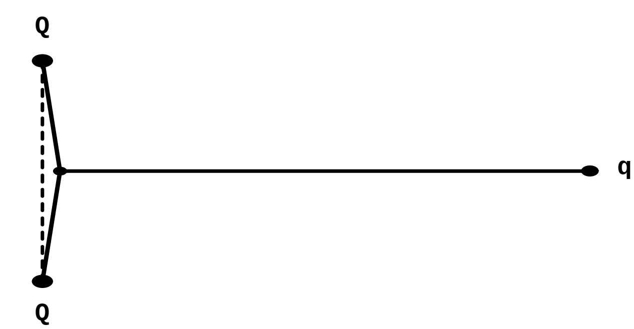

Figure 1.1. The representation of doubly heavy baryon QQq with the colored fields forming the strings between the heavy and light quarks, that destroys the picture of pair interactions and involves the additional 'centre-of-mass' point close to the centre of mass for the heavy-heavy system.

There is another point of view to the problem of three-quark bound states in QCD, i.e. the representation of quark-gluon string. In the string-like picture of doubly heavy baryon shown in Fig.1.1, the above conclusions on the structure of mass spectra for the doubly heavy baryons derived on the basis of pair interactions should be essentially modified. Indeed, to the moment we have to introduce the centre of string, which is very close to the centre of mass for the doubly heavy diquark. Furthermore, the light quark interacts with the doubly heavy diquark as a whole, i.e. with the string tension identical to that in the heavy-light mesons $Q\bar{q}$. Therefore, two different assumptions on the nature of interactions inside the doubly heavy baryons: pair interactions or string-like picture, result in a distinct variation of predictions on the mass spectra of these baryons for both the ground states and excitation levels. The only criterion testing the assumptions is provided by an experimental observation and measurements.

In this review we follow the approximation of doubly heavy diquark, which is quite reasonable as we have clarified in the discussion given above. To enforce this point we refer to the consideration

of doubly heavy baryon masses in the framework of QCD sum rules (see Chapter 2), which result in the masses of ground states in a good agreement with the estimates obtained in the potential approach with the factorization of doubly heavy diquark.

The qualitative picture for the forming of bound states in the system of $(QQ'q)$ is determined by the presence of two scales of distances, which are given by the size of QQ'-diquark subsystem, $r_{QQ'}$, in the anti-triplet color state as well as by the confinement scale, Λ_{QCD}, for the light quark q, so that

$$r_{QQ'} \cdot \Lambda_{QCD} \ll 1, \quad \Lambda_{QCD} \ll m_Q.$$

Under such conditions, the compact diquark QQ' looks like a static source approximated by the local colored QCD field interacting with the light quark. Therefore, we can use a set of reliable results in models of mesons with a single heavy quark, i.e. with a local static source belonging to the anti-triplet representation of $\mathrm{SU}(3)_c$ group. The successful approaches are the potential models [8] and the Heavy Quark Effective Theory (HQET) [9] in the framework of expansion in the inverse heavy quark mass. We apply the nonrelativistic quark model with the potential by Buchmüller–Tye [13]. Then *theoretically* we can talk on the rough approximation for the light quark. Indeed, since $m_q^{QCD} \ll \Lambda_{QCD}$ the light quark is relativistic. Nevertheless, we introduce the system with a finite number of degrees of freedom and an instantaneous interaction $V(\mathbf{r})$. This fact is a disadvantage because the confinement supposes the following: a) the generation of sea around the light quark, i.e. the presence of infinite number of gluons and quark-antiquark pairs, and b) the nonperturbative effects with the correlation time $\tau_{QCD} \sim 1/\Lambda_{QCD}$, that is beyond the potential approach. However, *phenomenologically* the introduction of constituent mass $m_q^{NP} \sim \Lambda_{QCD}$ as a basic parameter determining the interaction with the QCD condensates, allows us to successfully adjust the nonrelativistic potential model with a high accuracy ($\delta M \approx 30 - 40$ MeV) by fitting the existing experimental data, that makes the approach to be quite a reliable tool for the prediction of masses for the hadrons, containing the heavy and light quarks.

As for the diquark QQ', it is completely analogous to the heavy quarkonium $Q\bar{Q}'$ except the very essential peculiarities.

1. $(QQ')_{\bar{3}_c}$ is a system with the nonzero color charge.

2. For the quarks of the same flavor $Q = Q'$ it is necessary to take into account the Pauli principle for the identical fermions.

The second item turns out to forbid the sum of quark spins S=0 for the symmetric, spatial parity P-even wave functions of diquark, $\Psi_d(\mathbf{r})$ (the orbital momentum equals $L_d = 2n$, where $n = 0, 1, 2\dots$), as well as S=1 is forbidden for the anti-symmetric, -odd functions $\Psi_d(\mathbf{r})$ (i.e. $L_d = 2n + 1$). The nonzero color charge leads to two problems.

First, we cannot generally apply the confinement hypothesis on the form of potential (an infinite growth of energy with the increase of the system size) for the object under consideration. However, it is unpossible to imagine a situation, when a big colored object with a size $r > 1/\Lambda_{QCD}$ has a finite energy of self-action, and, to the same moment, it is confined inside a white hadron (the singlet over $\mathrm{SU}(3)_c$) with $r \sim 1/\Lambda_{QCD}$ due to the interaction with another colored source. In the framework of well-justified picture of the hadronic string, the tension of such string in the diquark with the external leg inside the baryons is only two times less than in the quark-antiquark pair inside the meson $q\bar{q}'$, and, hence, the energy of diquark linearly grows with the increase of its size. So, the effect analogous to the confinement of quarks takes place in the similar way. In the potential models we can suppose that the quark binding appears due to the effective single exchange by a

colored object in the adjoint representation of $SU(3)_c$ (the sum of scalar and vector exchanges is usually taken). Then, the potentials in the singlet $(q\bar{q}')$ and anti-triplet (qq') states differ by the factor of 1/2, that means the confining potential with the linear term in the QCD-motivated models for the heavy diquark $(QQ')_{\bar{3}_c}$. In the present chapter we use the nonrelativistic model with the Buchmüller–Tye potential for the diquark, too.

Second, in the singlet color state $(Q\bar{Q}')$ there are separate conservations of the summed spin S and the orbital momentum L, since the QCD operators for the transitions between the levels determined by these quantum numbers, are suppressed. Indeed, in the framework of multipole expansion in QCD [14], the amplitudes of chromo-magnetic and chromo-electric dipole transitions are suppressed by the inverse heavy quark mass, but in addition, the major reason is provided by the following: a) the necessity to emit a white object, i.e. at least two gluons, which results in the higher order in $1/m_Q$, and b) the projection to a real phase space in a physical spectrum of massive hadrons in contrast to the case of massless gluon. Furthermore, the probability of a hybrid state, say, the octet sybsystem $(Q\bar{Q}')$ and the additional gluon, i.e. the Fock state $|Q\bar{Q}'_{8_c}g\rangle$, is suppressed due to both the small size of system and the nonrelativistic motion of quarks (for a more strict consideration see ref. [10]). In the anti-triplet color state, the emission of a soft nonperturbative gluon between the levels determined by the spin S_d and the orbital momentum L_d of diquark, is not forbidden, if there are no some other no-go rules or small order-parameters. For the quarks of identical flavors inside the diquark, the Pauli principle leads to that the transitions are possible only between the levels, which either differ by the spin $(\Delta S_d = 1)$ and the orbital momentum $(\Delta L_d = 2n+1)$, instantaniously, or belong to the same set of radial and orbital excitations with $\Delta L_d = 2n$. Therefore, the transition amplitudes are suppressed by a small recoil momentum of diquark in comparison with its mass. The transition operator changing the diquark spin as well as its orbital momentum, has the higher order of smallness because of either the additional factor of $1/m_Q$ or the small size of diquark. These suppressions lead to the existence of quasi-stable states with the quantum numbers of S_d and L_d. In the diquark composed by the quarks of different flavors, bc, the QCD operators of dipole transitions with the single emission of soft gluon are not forbidden, so that the lifetimes of levels can be about the times for the forming of bound states or with the inverse distances between the levels themselves. Then, we cannot insist on the appearance of excitation system for such the diquark with definite quantum numbers of the spin and orbital momentum[3].

Thus, in the present review we explore the presence of two physical scales in the form of factorization for the wave functions of the heavy diquark and light constituent quark. So, in the framework of nonrelativistic quark model the problem on the calculation of mass spectrum and characteristics of bound states in the system of doubly heavy baryon is reduced to two standard problems on the study of stationary levels of energy in the system of two bodies. After that, we take into account the relativistic corrections dependent of the quark spins in two subsystems under consideration. The natural boundary for the region of stable states in the doubly heavy system can be assigned to the threshold energy for the decay into a heavy baryon and a heavy meson. As was shown in [15], the appearance of such threshold in different systems can be provided by the existence of an universal characteristics in QCD, a critical distance between the quarks. At distances greater than the critical separation, the quark-gluon fields become unstable, i.e. the generation of valence quark-antiquark pairs from the sea takes place. In other words, the hadronic string having a length greater than the critical one, decay into the strings of smaller sizes with a high probability close to unit. In the

[3] In other words, the presence of gluon field inside the baryon Ξ_{bc} leads to the transitions between the states with the different excitations of diquark, like $|bc\rangle \to |bcg\rangle$ with $\Delta S_d = 1$ or $\Delta L_d = 1$, which are not suppressed.

framework of potential approach this effect can be taken into account by that we will restrict the consideration of excited diquark levels by the region, wherein the size of diquark is less than the critical distance, $r_{QQ'} < r_c \approx 1.4 - 1.5$ fm. Furthermore, the model with the isolated structure of diquark looks to be reliable, just if the size of diquark is less than the distance to the light quark $r_{QQ'} < r_l$.

The peculiarity of quark-diquark picture for the doubly heavy baryon is the possibility of mixing between the states of higher diquark excitations, possessing the different quantum numbers, because of the interaction with the light quark. Then it is difficult to assign some definite quantum numbers to such excitations. We will discuss the mechanism of this effect.

In Section 1.1 we describe a general procedure for the calculation of masses for the doubly heavy baryons in the framework of assumptions drawn above. We take into account the spin-dependent corrections to the potential motivated in QCD. The results of numerical estimates are presented in Section 1.2, and, finally, our conclusions are discussed in the end of this chapter.

1.1. Nonrelativistic potential model

As we have mentioned in the Introduction, we solve the problem on the calculation of mass spectra of baryons containing two heavy quarks, in two steps. First, we compute the energy levels of diquark. Second, we consider the two-body problem for the light quark interacting with the point-like diquark having the mass obtained in the first step. In accordance with the effective expansion of QCD in the inverse heavy quark mass, we separate two stages of such the calculations. So, the nonrelativistic Schrödinger equation with the model potential motivated by QCD, is solved numerically. After that, the spin-dependent corrections are introduced as perturbations suppressed by the quark masses.

1.1.1. Potential

The potential of static heavy quarks illuminates the most important features of QCD dynamics: the asymptotic freedom and confinement. In the leading order of perturbative QCD at short distances and with a linear confining term in the infrared region, the potential of static heavy quarks was considered in the Cornell model [16], incorporating the simple superposition of both asymptotic limits (the effective coulomb and string-like interactions). The observed heavy quarkonia posed in the intermediate distances, where both terms are important for the determination of mass spectra (see Fig. 1.2). So, the phenomenological approximations of potential (logarithmic one [17] and power law [18]), taking into account the regularities of such the spectra, were quite successful [8].

The quantities more sensitive to the global properties of potential are the wave functions at the origin as related to the leptonic constants and production rates. So, the potentials consistent with the asymptotic freedom to one and two loops as well as the linear confinement were proposed by Richardson [19], Buchmüller and Tye [13], respectively.

In QCD the static potential is defined in a manifestly gauge invariant way by means of the vacuum expectation value of a Wilson loop [20],

$$\begin{aligned} V(r) &= -\lim_{T\to\infty} \frac{1}{iT} \ln\langle \mathcal{W}_\Gamma \rangle \,, \\ \mathcal{W}_\Gamma &= \widetilde{\mathrm{tr}}\, \mathcal{P} \exp\left(ig \oint_\Gamma dx_\mu A^\mu \right) \,. \end{aligned} \tag{1.1}$$

Here, Γ is taken as a rectangular loop with time extension T and spatial extension r. The gauge fields A_μ are path-ordered along the loop, while the color trace is normalized according to $\widetilde{\mathrm{tr}}(..) =$

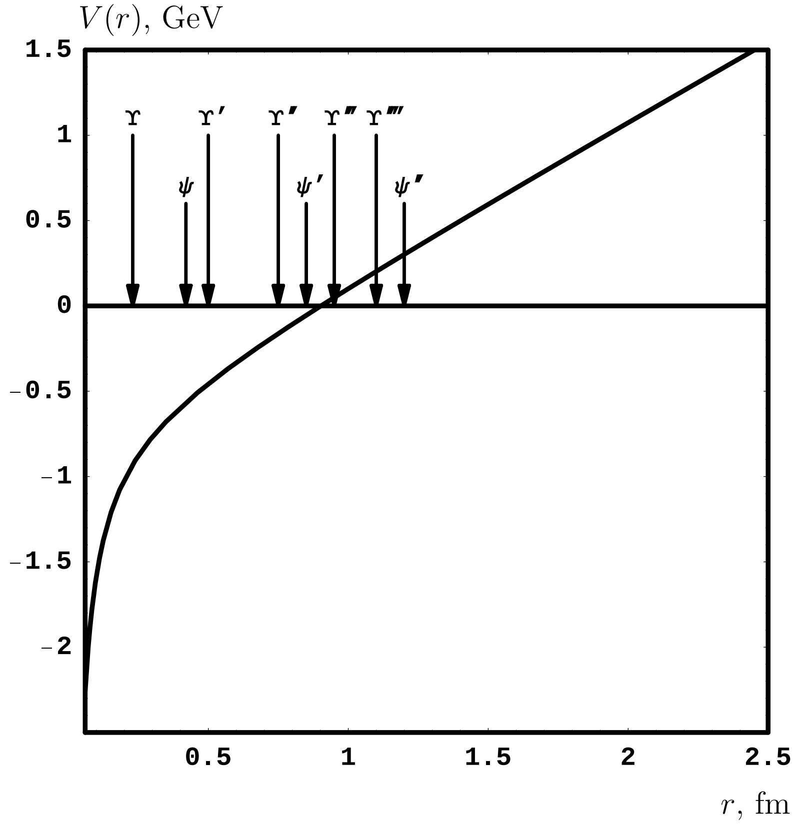

Figure 1.2. The Corenell model of static potential and sizes of observed heavy quarkonia with charmed quarks (the family ψ) and bottom quarks (the family Υ).

$\mathrm{tr}(..)/\mathrm{tr}\,\mathbb{1}$. This definition corresponds to the calculation of effective action for the case of two external sources fixed at a distance r during an infinitely long time period T, so that the time-ordering coincides with the path-ordering. Moreover, the contribution into the effective action by the path parts, where the charges have been separated to the finite distance during a finite time, can be neglected in comparison with the infinitely growing term of $V(r)\cdot T$. Let us emphasize that the defined static potential is, by construction, the renormalization invariant quantity, since the action, by definition, doe not depend on the normalization point.

Generally, one introduces the V scheme of QCD coupling constant by the definition of QCD potential of static quarks in momentum space as follows:

$$V(\mathbf{q}^2) = -C_F \frac{4\pi\alpha_{\rm V}(\mathbf{q}^2)}{\mathbf{q}^2}, \tag{1.2}$$

so that for the such-way introduced value of $\alpha_{\rm V}$ one can derive some results at large virtualities in the perturbative QCD as well as at low transfer momenta in the approximation of linear term in the potential confining the quarks.

In this section, first, we discuss two regimes for the QCD forces between the static heavy quarks: the asymptotic freedom and confinement. Then we follow the method by Buchmüller and Tye and formulate how these regimes can be combined in a unified β function for $\alpha_{\rm V}$ obeyed both limits of small and large QCD couplings.

1.1.2. Perturbative results at short distances

Technically, using a given scheme of regularization, say, $\overline{\mathrm{MS}}$, one has to calculate the perturbative expansion for the potential of static quarks. This potential can be written down as the coulomb one with the running coupling constant in the so-called V scheme. Thus, the perturbative calculations provide us with the matching of $\overline{\mathrm{MS}}$ scheme with V-one. The calculations with the n loop running of $\alpha_s^{\overline{\mathrm{MS}}}$ requires the $n-1$ loop matching to α_{V}. Note, that initial two coefficients of corresponding β functions are scheme and gauge independent, while others generally depend. The V scheme is defined for the observed quantity, that implies its β function to be gauge invariant.

In the perturbative QCD the quantity α_{V} can be matched with $\alpha_{\overline{\mathrm{MS}}}$

$$\alpha_{\mathrm{V}}(\mathbf{q}^2) \;=\; \alpha_{\overline{\mathrm{MS}}}(\mu^2)\sum_{n=0}^{\infty}\tilde{a}_n(\mu^2/\mathbf{q}^2)\left(\frac{\alpha_{\overline{\mathrm{MS}}}(\mu^2)}{4\pi}\right)^n = \alpha_{\overline{\mathrm{MS}}}(\mathbf{q}^2)\sum_{n=0}^{\infty}a_n\left(\frac{\alpha_{\overline{\mathrm{MS}}}(\mathbf{q}^2)}{4\pi}\right)^n. \tag{1.3}$$

Two loop results for the β function and the one loop matching condition for the potential were available to the moment of Buchmüller-Tye publication. Recently, the progress in calculations has provided us with the two loop matching of V and $\overline{\mathrm{MS}}$ schemes [21, 22], that can be combined with the three loop running of $\alpha_s^{\overline{\mathrm{MS}}}$. At present, in expansion (1.3) the coefficients of tree approximation a_0, the one loop contribution a_1 and new results for the two-loop term a_2 (see [21, 22]) are known.

After the introduction of $\mathfrak{a} = \frac{\alpha}{4\pi}$, the β function is actually defined by

$$\frac{d\mathfrak{a}(\mu^2)}{d\ln\mu^2} = \beta(\mathfrak{a}) = -\sum_{n=0}^{\infty}\beta_n\cdot\mathfrak{a}^{n+2}(\mu^2), \tag{1.4}$$

so that $\beta_{0,1}^{\mathrm{V}} = \beta_{0,1}^{\overline{\mathrm{MS}}}$ and $\beta_2^{\mathrm{V}} = \beta_2^{\overline{\mathrm{MS}}} - a_1\beta_1^{\overline{\mathrm{MS}}} + (a_2 - a_1^2)\beta_0^{\overline{\mathrm{MS}}}$.

The Fourier transform results in the position-space potential [21]

$$\begin{aligned} V(r) \;=\; & -C_F\frac{\alpha_{\overline{\mathrm{MS}}}(\mu^2)}{r}\Bigg[1 + \frac{\alpha_{\overline{\mathrm{MS}}}(\mu^2)}{4\pi}\Big(2\beta_0\ln(\mu r') + a_1\Big) + \\ & + \Big(\frac{\alpha_{\overline{\mathrm{MS}}}(\mu^2)}{4\pi}\Big)^2\Big(\beta_0^2(4\ln^2(\mu r') + \frac{\pi^2}{3}) + 2(\beta_1 + 2\beta_0 a_1)\ln(\mu r') + a_2\Big)\Bigg], \end{aligned} \tag{1.5}$$

with $r' \equiv r\exp(\gamma_E)$. Defining the new running coupling constant, depending on the distance,

$$V(r) = -C_F\frac{\bar{\alpha}_{\mathrm{V}}(1/r^2)}{r}. \tag{1.6}$$

we can calculate its β function from (1.5), so that [21]

$$\bar{\beta}_2^{\mathrm{V}} = \beta_2^{\mathrm{V}} + \frac{\pi^2}{3}\beta_0^3, \tag{1.7}$$

and the minor coefficients $\bar{\beta}_{0,1}^{\mathrm{V}}$ are equal to the values independent of the scheme. Note that the perturbative potential (1.5), by construction, is independent of normalization point, i.e. it is the renormalization group invariant. However, in the problem under consideration the truncation of perturbative expansion, wherein the coefficients do not decrease[4], leads to a strong custodial

[4]Moreover, according to the investigations of renormalon, the coefficients in the series of perturbation theory for the potential increase in the factorial power, so that the series has a meaning of asymptotic one.

dependence on the normalization point. So, putting the normalization point μ in the region of charned quark mass, we find that the two-loop potential with the three-loop running coupling constant $\alpha_s^{\overline{\rm MS}}$ has an unremovable additive shift depending on μ. This shift has variation in wide limits. This fact illuminates the presence of infrared singularity in the coupling constant of QCD, so that the μ-dependent shift in the potential energy has the form of a pole posed at Λ_{QCD} [24].

Thus, in order to avoid the ambiguity of static potential in QCD we have to deal with infrared stable quantities. The motivation by Buchmüller and Tye was to write down the β function of $\alpha_{\rm V}$ consistent with two known asymptotic regimes at short and long distances. They proposed the function, which results in the effective charge determined by two parameters, only: the perturbative parameter is the scale in the running of coupling constant at large virtualities and the nonperturbative parameter is the string tension. The necessary inputs are the coefficients of β function. The parameters of potential by Buchmüller and Tye were fixed by fitting the mass spectra of charmonium and bottomonium [23]. Particularly, in such the phenomenological approach the scale $\Lambda_{\overline{\rm MS}}^{n_f=4} \approx 510$ MeV was determined. It determines the asymptotic behaviour of coupling constant at latge virtualities in QCD. This value is in a deep contradiction with the current data on the QCD coupling constant $\alpha_s^{\overline{\rm MS}}$ [23]. In addition, one can easily find that the three-loop coefficient $\beta_2^{\rm V}$ for the β function suggested by Buchmüller and Tye is not correct even by its sign and absolute value in comparison with the exact coefficient recently calculated in [21, 22].

Thus, the modification of Buchmüller–Tye (BT) potential of static quarks as dictated by the current status of perturbative calculations is of great interest.

To normalize the couplings in deep perturbative region, we use (1.3) at $\mathbf{q}^2 = m_Z^2$.

1.1.3. The quark confinement

The nonperturbative behaviour of QCD forces between the static heavy quarks at long distances r is usually represented by the linear potential (see discussion in ref. [25])

$$V^{\rm conf}(r) = k \cdot r, \tag{1.8}$$

which corresponds to the square-law limit for the Wilson loop.

We can represent this potential in terms of constant chromo-electric field between the sources posed in the fundamental representation of SU(N_c). So, in the Fock-Schwinger gauge of fixed point $x_\mu \cdot A^\mu(x) = 0$, we can represent the gluon field by means of strength tensor $A_\mu(x) \approx \frac{1}{2} x^\nu G_{\mu\nu}(0)$, so that for the static quarks separated by the distance $\mathbf{r}$ we have $\bar{Q}_i(0)\; G^a_{m0}(0)\; Q_j(0) = \frac{\mathbf{r}_m}{r}\; E\; T^a_{ij}$, where the heavy quark fields are normalized to unit. Then, the confining potential is written down as

$$V^{\rm conf}(r) = \frac{1}{2} g_s\, C_F\, E \cdot r.$$

Supposing, that the same strength of the field is responsible for the forming of gluon condensate (see Fig. 1.3), and introducing the colored sources n_i, which have to be averaged in the vacuum, we can easily find

$$\langle G^2_{\mu\nu}\rangle = 4\, C_F\, E^2 \langle \bar{n} n\rangle.$$

For the linear term in the potential, the consideration in [24] leads to

$$k = \frac{\pi}{2\sqrt{N_c}}\, C_F \sqrt{\langle \frac{\alpha_s}{\pi} G^2_{\mu\nu}\rangle}. \tag{1.9}$$

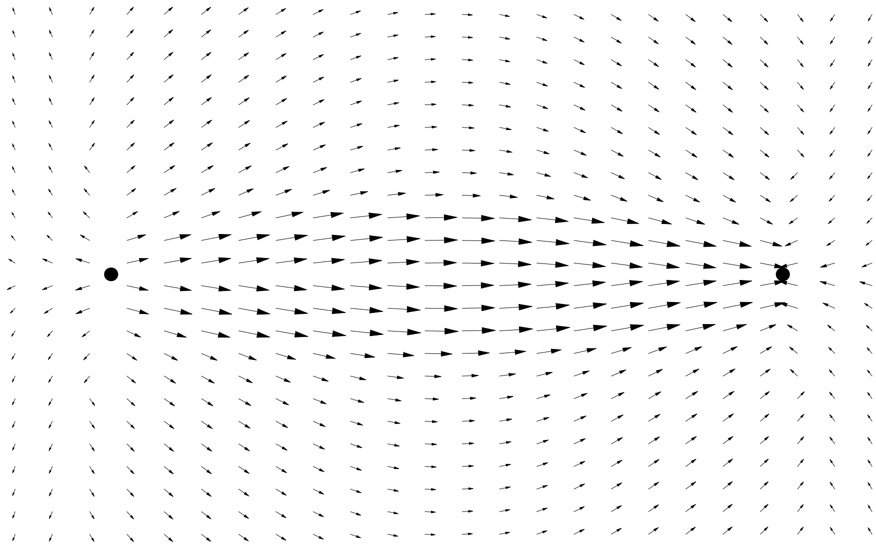

Figure 1.3. Under the action of charged sources, the vacuum chromo-electric field is aligned along the axis connecting the heavy quarks.

The k term is usually represented through a parameter α'_P as

$$k = \frac{1}{2\pi\alpha'_P}.$$

Following Buchmüller and Tye, we put $\alpha'_P = 1.04$ GeV^{-2}. This value of tension, that is related with a slope of Regge trajectories, can be compared with the estimate following from (1.9). At $\langle\frac{\alpha_s}{\pi}G^2_{\mu\nu}\rangle = (1.6 \pm 0.1)\cdot 10^{-2}$ GeV^4 [7] we have found

$$\alpha'_P = 1.04 \pm 0.03 \;\; \text{GeV}^{-2},$$

which is in a good agreement with the well known value of Regge trajectory slope.

The form of (1.8) corresponds to the limit, when at low virtualities $\mathbf{q}^2 \to 0$ the coupling α_V tends to

$$\alpha_\mathrm{V}(\mathbf{q^2}) \to \frac{K}{\mathbf{q^2}},$$

so that

$$\frac{d\alpha_\mathrm{V}(\mathbf{q^2})}{\ln\mathbf{q^2}} \to -\alpha_\mathrm{V}(\mathbf{q^2}), \tag{1.10}$$

which gives the confinement asymptotics for the β_V function.

1.1.4. Unified β function and potential

Buchmüller and Tye proposed the procedure for the reconstruction of β function in the whole region of charge variation by the known limits of asymptotic freedom to a given order in α_s and confinement regime. Generalizing their method, the β_{PT} function found in the framework of asymptotic perturbative theory (PT) to three loops, is transformed to the β function of effective charge as

follows

$$
\begin{aligned}
\frac{1}{\beta_{\rm PT}(\mathfrak{a})} &= -\frac{1}{\beta_0\mathfrak{a}^2}+\frac{\beta_1+\left(\beta_2^{\rm V}-\frac{\beta_1^2}{\beta_0}\right)\mathfrak{a}}{\beta_0^2\mathfrak{a}} \Longrightarrow \\
\frac{1}{\beta(\mathfrak{a})} &= -\frac{1}{\beta_0\mathfrak{a}^2\left(1-\exp\left[-\frac{1}{\beta_0\mathfrak{a}}\right]\right)}+\frac{\beta_1+\left(\beta_2^{\rm V}-\frac{\beta_1^2}{\beta_0}\right)\mathfrak{a}}{\beta_0^2\mathfrak{a}}\exp\left[-\frac{l^2\mathfrak{a}^2}{2}\right],
\end{aligned} \tag{1.11}
$$

where the exponential factor in the second term contributes to the next-to-next-to-leading order at $\mathfrak{a}\to 0$. This function has the essential peculiarity at $\mathfrak{a}\to 0$, so that the expansion is the asymptotic series in $\mathfrak{a}$. At $\mathfrak{a}\to\infty$ the β function tends to the confinement limit represented in (1.10). Remember that the one and two-loop static potentials matched with the linear term of confinement lead to the contradiction with the value of QCD coupling constant extracted at the scale of Z boson mass if we fit the mass spectra of heavy quarkonia in such potentials. We will show that the static potential in the three-loop approximation results in the consistent value of QCD coupling constant at large virtualities.

In the perturbative limit the usual solution for the running coupling constant

$$
\begin{aligned}
\mathfrak{a}(\mu^2) = \frac{1}{\beta_0\ln\frac{\mu^2}{\Lambda^2}} \quad & \left[1-\frac{\beta_1}{\beta_0^2}\frac{1}{\ln\frac{\mu^2}{\Lambda^2}}\ln\ln\frac{\mu^2}{\Lambda^2}+\right. \\
& \left.\frac{\beta_1^2}{\beta_0^4}\frac{1}{\ln^2\frac{\mu^2}{\Lambda^2}}\left(\ln^2\ln\frac{\mu^2}{\Lambda^2}-\ln\ln\frac{\mu^2}{\Lambda^2}-1+\frac{\beta_2^{\rm V}\beta_0}{\beta_1^2}\right)\right],
\end{aligned} \tag{1.12}
$$

is valid. Using the asymptotic limit of (1.12), one can get the equation

$$
\ln\frac{\mu^2}{\Lambda^2} = \frac{1}{\beta_0\mathfrak{a}(\mu^2)}+\frac{\beta_1}{\beta_0^2}\ln\beta_0\mathfrak{a}(\mu^2)+\int_0^{\mathfrak{a}(\mu^2)}dx\left[\frac{1}{\beta_0x^2}-\frac{\beta_1}{\beta_0^2x}+\frac{1}{\beta(x)}\right], \tag{1.13}
$$

which can be easily integrated out, so that we get an implicit solution for the charge depending on the scale. The implicit equation can be inverted by the iteration procedure, so that well approximated solution has the form

$$
\mathfrak{a}(\mu^2) = \frac{1}{\beta_0\ln\left(1+\eta(\mu^2)\frac{\mu^2}{\Lambda^2}\right)}, \tag{1.14}
$$

where $\eta(\mu^2)$ is expressed through the coefficients of perturbative β function and parameter l, which is related to the slope of Regge trajectories and the integration constant, the scale Λ, by the relation

$$
\ln 4\pi^2 C_F\alpha'_P\Lambda^2 = \ln\beta_0+\frac{\beta_1}{2\beta_0^2}\left(\gamma_E+\frac{l^2}{2\beta_0^2}\right)-\frac{\beta_2^{\rm V}\beta_0-\beta_1^2}{\beta_0^3}\frac{\sqrt{\frac{\pi}{2}}}{l}, \tag{1.15}
$$

which completely fixes the parameters of β function and the charge in terms of scale Λ and slope α'_P.

1.1.5. Setting the scales

As we have already mentioned the slope of Regge trajectories, determining the linear part of potential, is fixed as $\alpha'_P = 1.04$ GeV^{-2}. We use also the measured value of QCD coupling constant [23] and pose

$$
\alpha_s^{\overline{\rm MS}}(m_Z^2) = 0.123,
$$

as the basic input of the potential. Then, we evaluate $\alpha_{\rm V}(m_Z^2) \approx 0.1306$, and put it as the normalization point for $\mathfrak{a}(m_Z^2) = \alpha_{\rm V}(m_Z^2)/(4\pi)$. Further, we find the values of Λ for the effective charge, depending on the number of active flavors [24]. After determining the momentum space dependence of the charge, we perform the Fourier transform to get

$$V(r) = k \cdot r - \frac{8C_F}{r} u(r), \tag{1.16}$$

with the function

$$u(r) = \int_0^\infty \frac{dq}{q} \left(\mathfrak{a}(q^2) - \frac{K}{q^2} \right) \sin(q \cdot r),$$

which is calculated numerically at $r > 0.01$ fm, while at short distances the behaviour of potential is purely perturbative, so that at $r < 0.01$ fm we put

$$V(r) = -C_F \, \frac{\bar{\alpha}_{\rm V}(1/r^2)}{r}, \tag{1.17}$$

where the running $\bar{\alpha}_{\rm V}(1/r^2)$ is given by eq.(1.12) with the appropriate value of $\bar{\beta}_2^{\rm V}$ at $n_f = 5$, and with the matching with the potential (1.16) at $r_s = 0.01$ fm, where we have found $\bar{\alpha}_{\rm V}(1/r_s^2) = 0.22213$, which implies $\Lambda^{\overline{\rm V}}_{n_f=5} = 617.42$ MeV.

Thus, we have completely determined the potential of static heavy quarks in QCD with the three-loop running of coupling constant. In Fig. 1.4 we present the potential versus the distance between the quarks. As we can see the potential is very close to what was obtained in the Cornell model in the phenomenological manner by fitting the mass spectra of heavy quarkonia. We can draw the conclusion that accepting the normalization by the value of QCD coupling constant at the virtuality $q^2 = m_Z^2$ and using the three-loop evolution for the effective charge incorporating the confinement with the linear term, we have got the static potential consistent with the phenomenological models and, hence, with the calculations of mass spectra for the heavy quarkonia in the nonrelativistic approximation.

Such the consistency of potential with the parameters of QCD has become possible due to the fact that in the two-loop approximation for the coulomb potential the calculations have led to the essential corrections to the β function of effective charge, so that $\Delta\beta/\beta \sim 10\%$. This correction is important for the determination of critical values of charge, i.e. the value in the intermediate region between the perturbative and nonperturbative regimes. Moreover, the two-loop matching condition and the three-loop running of coupling constant normalized by the data at the high energy of m_Z determine the region of energetic scale for changing the regimes mentioned above. This scale strongly correlates with the data on the mass spectra of heavy quarkonia. Indeed, it is connected with the splitting of masses between the $1S$ and $2S$ levels. We stress that the two-loop improvement gives the correct normalization of effective coulomb exchange at the distances characteristic for the average separation between the heavy quarks inside the heavy quarkonium and determines the evolution at short distances $r < 0.08$ fm, that is important in the calculations of leptonic constants related with the wave functions at the origin.

The analysis of potential for the static quarks in the calculations of both the mass spectra for the heavy quarkonia and the leptonic constants for the vector states is presented in [24], where the heavy quark masses are determined in the potential approach.

We emphasize that the potential by Buchmüller and Tye was obtained under fitting the experimental mass spectra for the heavy quarkonia, and it is numerically very close to the static potential

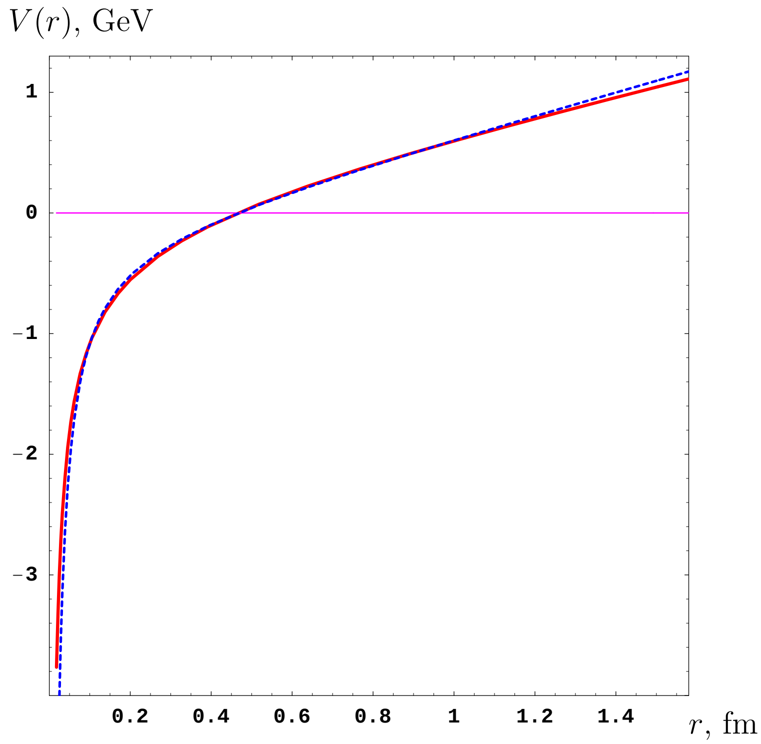

Figure 1.4. The potential of static heavy quarks in QCD (solid line) in comparison with the Cornell model (dashed line) (up to an additive shift of energy scale).

under consideration consistent with the normalization of QCD coupling constant at large virualities. Therefore, the Buchmüller–Tye potential continues to keep its phenomenological usage for the calculations of mass levels for the hadrons with c and b quarks with the accuracy about 40-70 MeV, that is a systematic uncertainty of potential approach.

1.1.6. System of levels

Following ref. [26], in the potential model we use the Buchmüller–Tye ansatz, which takes into account the coulomb corrections at short distances with the running coupling constant in two loops, while at large distances the interaction energy linearly increases, that provides the confinement. In the anti-triplet quark state we introduce the factor of 1/2 because of the color structure of bound quark-quark state. For the interaction of diquark with the light constituent quark, the corresponding factor is equal to unit.

As was shown in [27], the nonperturbative constituent term introduced into the mass of nonrelativistic quark, exactly coincides with the additive constant, subtracted from the coulomb potential.

Thus, we extract the masses of heavy quarks by fitting the real spectra of charmonium and bottomonium,

$$m_c = 1.486 \text{ GeV}, \quad m_b = 4.88 \text{ GeV}, \tag{1.18}$$

so that the mass of the level in the heavy quarkonium has been calculated as, say, $M(\bar{c}) = 2m_c + E$, where E is the energy of stationary Schrödinger equation with the model potential V. Then, we have supposed that the mass of meson with a single heavy quark is equal to $M(Q\bar{q}) = m_Q + m_q + E$, and $E = \langle T\rangle + \langle V - \delta V\rangle$, whereas the additive term in the potential is introduced because the constituent mass of light quark is determined as a part of interaction energy $\delta V = m_q$. In accordance with

fitting the masses of heavy mesons, we get $m_q = 0.385$ GeV.

Table 1.1. The spectrum of bb-diquark levels without spin-dependent splittings: masses and mean-squared radii.

diquark level	, GeV	$\langle r^2\rangle^{1/2}$, fm	diquark level	, GeV	$\langle r^2\rangle^{1/2}$, fm
1S	9.74	0.33	2P	9.95	0.54
2S	10.02	0.69	3P	10.15	0.86
3S	10.22	1.06	4P	10.31	1.14
4S	10.37	1.26	5P	10.45	1.39
5S	10.50	1.50	6P	10.58	1.61
3D	10.08	0.72	4D	10.25	1.01
5D	10.39	1.28	6D	10.53	1.51
4F	10.19	0.87	5F	10.34	1.15
6F	10.47	1.40	5G	10.28	1.01
6G	10.42	1.28	6M	10.37	1.15

Table 1.2. The spectrum of bc-diquark levels without spin-dependent splittings: masses and mean-squared radii.

diquark level	, GeV	$\langle r^2\rangle^{1/2}$, fm	diquark level	, GeV	$\langle r^2\rangle^{1/2}$, fm
1S	6.48	0.48	3P	6.93	1.16
2S	6.79	0.95	4P	7.13	1.51
3S	7.01	1.33	3D	6.85	0.96
2P	6.69	0.74	4D	7.05	1.35
4F	6.97	1.16	5F	7.16	1.52
5G	7.09	1.34	6H	7.19	1.50

The results of calculations for the energy levels in the Schrödinger equation with the Buchmüller–Tye potential for the various diquark systems are presented in Tables 1.1–1.3, while the characteristics of corresponding wave functions are shown in Tables 1.4–1.6.

We have checked that with a good accuracy *the binding energy and the wave function of light quark practically do not depend on the flavors of heavy quarks.* Indeed, large values of diquark masses give small contributions into the reduced masses. This fact leads to small corrections to the wave functions in the Schrödinger equation.

So, for the states lying below the threshold of doubly heavy baryon decay into the heavy baryon and heavy meson, the energies of levels of light constituent quark are equal to

$$E(1s) = 0.38 \text{ GeV},\ \ E(2s) = 1.09 \text{ GeV},\ \ E(2p) = 0.83 \text{ GeV},$$

where the energy has been defined as the sum of light quark constituent mass and eigen-value of Schrödinger equation. In HQET the value of $\bar{\Lambda} = E(1s)$ is generally introduced. Then we can draw a conclusion that our estimate of $\bar{\Lambda}$ is in a good agreement with calculations in other approaches.

Table 1.3. The spectrum of cc-diquark levels without spin-dependent splittings: masses and mean-squared radii.

diquark level	, GeV	$\langle r^2 \rangle^{1/2}$, fm	diquark level	, GeV	$\langle r^2 \rangle^{1/2}$, fm
1S	3.16	0.58	3P	3.66	1.36
2S	3.50	1.12	4P	3.90	1.86
3S	3.76	1.58	3D	3.56	1.13
2P	3.39	0.88	4D	3.80	1.59

Table 1.4. The characteristics of radial wave function for the bb-diquark: $R_{d(ns)}(0)$ (GeV$^{3/2}$), $R'_{d(np)}(0)$ (GeV $^{5/2}$).

nL	$R_{d(ns)}(0)$	nL	$R'_{d(np)}(0)$
1S	1.346	2P	0.479
2S	1.027	3P	0.539
3S	0.782	4P	0.585
4S	0.681	5P	0.343

Table 1.5. The characteristics of radial wave function for the bc-diquark: $R_{d(ns)}(0)$ (GeV$^{3/2}$), $R'_{d(np)}(0)$ (GeV $^{5/2}$).

nL	$R_{d(ns)}(0)$	nL	$R'_{d(np)}(0)$
1S	0.726	2P	0.202
2S	0.601	3P	0.240

Table 1.6. The characteristics of radial wave function for the cc-diquark: $R_{d(ns)}(0)$ (GeV$^{3/2}$), $R'_{d(np)}(0)$ (GeV $^{5/2}$).

nL	$R_{d(ns)}(0)$	nL	$R'_{d(np)}(0)$
1S	0.530	2P	0.128
2S	0.452	3P	0.158

This fact confirms the reliability of such the phenomenological predictions. For the light quark radial wave functions at the origin we find

$$R_{1S}(0) = 0.527 \text{ GeV}^{3/2},\;\; R_{2S}(0) = 0.278 \text{ GeV}^{3/2},\;\; R'_{2P}(0) = 0.127 \text{ GeV}^{5/2}.$$

The analogous characteristics of bound states of the c-quark interacting with the bb-diquark, are equal to

$$E(1s) = 1.42 \text{ GeV},\;\; E(2s) = 1.99 \text{ GeV},\;\; E(2p) = 1.84 \text{ GeV},$$

with the wave functions

$$R_{1S}(0) = 1.41 \text{ GeV}^{3/2},\;\; R_{2S}(0) = 1.07 \text{ GeV}^{3/2},\;\; R'_{2P}(0) = 0.511 \text{ GeV}^{5/2}.$$

For the binding energy of strange constituent quark we add the current mass $m_s \approx 100 - 150$ MeV.

1.1.7. Spin-dependent corrections

According to [28], we introduce the spin-dependent corrections causing the splitting of nL-levels of diquark as well as in the system of light constituent quark and diquark ($n = n_r + L + 1$ is the principal number, n_r is the number of radial excitation, L is the orbital momentum). For the heavy diquark containing the identical quarks we have

$$\begin{aligned} V^{(d)}_{SD}(\mathbf{r}) &= \frac{1}{2}\left(\frac{\mathbf{L_d}\cdot\mathbf{S_d}}{2m_Q^2}\right)\left(-\frac{dV(r)}{rdr} + \frac{8}{3}\alpha_s\frac{1}{r^3}\right) \\ &+\frac{2}{3}\alpha_s\frac{1}{m_Q^2}\frac{\mathbf{L_d}\cdot\mathbf{S_d}}{r^3} + \frac{4}{3}\alpha_s\frac{1}{3m_Q^2}\mathbf{S}_{Q1}\cdot\mathbf{S}_{Q2}[4\pi\delta(\mathbf{r})] \\ &-\frac{1}{3}\alpha_s\frac{1}{m_Q^2}\frac{1}{4\mathbf{L_d}^2-3}[6(\mathbf{L_d}\cdot\mathbf{S_d})^2 + 3(\mathbf{L_d}\cdot\mathbf{S_d}) - 2\mathbf{L_d}^2\mathbf{S_d}^2]\frac{1}{r^3}, \end{aligned} \tag{1.19}$$

where $\mathbf{L_d}$, $\mathbf{S_d}$ are the orbital momentum in the diquark system and the summed spin of quarks composing the diquark, respectively. Taking into account the interaction with the light constituent quark gives ($\mathbf{S} = \mathbf{S_d} + \mathbf{S_l}$)

$$\begin{aligned} V^{(l)}_{SD}(\mathbf{r}) &= \frac{1}{4}\left(\frac{\mathbf{L}\cdot\mathbf{S_d}}{2m_Q^2} + \frac{2\mathbf{L}\cdot\mathbf{S_l}}{2m_l^2}\right)\left(-\frac{dV(r)}{rdr} + \frac{8}{3}\alpha_s\frac{1}{r^3}\right) \\ &+\frac{1}{3}\alpha_s\frac{1}{m_Q m_l}\frac{(\mathbf{L}\cdot\mathbf{S_d} + \mathbf{2L}\cdot\mathbf{S_l})}{r^3} + \frac{4}{3}\alpha_s\frac{1}{3m_Q m_l}(\mathbf{S_d}+\mathbf{L_d})\cdot\mathbf{S_l}[4\pi\delta(\mathbf{r})] \\ &-\frac{1}{3}\alpha_s\frac{1}{m_Q m_l}\frac{1}{4\mathbf{L}^2-3}[6(\mathbf{L}\cdot\mathbf{S})^2 + 3(\mathbf{L}\cdot\mathbf{S}) - 2\mathbf{L}^2\mathbf{S}^2 \\ &-6(\mathbf{L}\cdot\mathbf{S_d})^2 - 3(\mathbf{L}\cdot\mathbf{S_d}) + 2\mathbf{L}^2\mathbf{S_d}^2]\frac{1}{r^3}, \end{aligned} \tag{1.20}$$

where the first term corresponds to the relativistic correction to the effective scalar exchange, and other terms appear because of corrections to the effective single-gluon exchange with the coupling constant α_s.

The value of effective parameter α_s can be determined in the following way. The splitting in the S-wave heavy quarkonium $(Q_1\bar{Q}_2)$ is given by the expression

$$\Delta M(ns) = \frac{8}{9}\alpha_s \frac{1}{m_1 m_2}|R_{nS}(0)|^2, \tag{1.21}$$

where $R_{nS}(r)$ is the radial wave function of quarkonium. From the experimental data on the system of $c\bar{c}$

$$\Delta M(1S, c\bar{c}) = 117 \pm 2 \text{ MeV}, \tag{1.22}$$

and $R_{1S}(0)$ calculated in the model, we can determine $\alpha_s(\Psi)$.

Let us take into account the dependence of this parameter on the reduced mass of the system, μ. In the framework of one-loop approximation for the running coupling constant of QCD we have

$$\alpha_s(p^2) = \frac{4\pi}{b \cdot \ln(p^2/\Lambda^2_{QCD})}, \tag{1.23}$$

whereas $b = 11 - 2n_f/3$ and $n_f = 3$ at $p^2 < m_c^2$. From the phenomenology of potential models we know that the average kinetic energy of quarks in the bound state practically does not depend on the flavors of quarks, and it is given by the values

$$\langle T_d \rangle \approx 0.2 \text{ GeV}, \tag{1.24}$$

$$\langle T_l \rangle \approx 0.4 \text{ GeV}, \tag{1.25}$$

for the anti-triplet and singlet color states, correspondingly. Substituting the definition of the nonrelativistic kinetic energy

$$\langle T \rangle = \frac{\langle p^2 \rangle}{2\mu}, \tag{1.26}$$

we get

$$\alpha_s(p^2) = \frac{4\pi}{b \cdot \ln(2\langle T \rangle \mu/\Lambda^2_{QCD})}, \tag{1.27}$$

whereas numerically $\Lambda_{QCD} \approx 113$ MeV.

For the identical quarks inside the diquark, the scheme of LS-coupling well known for the corrections in the heavy quarkonium, is applicable. Otherwise, for the interaction with the light quark we use the scheme of jj-coupling (here, $\mathbf{LS_l}$ is diagonal at the given $\mathbf{J_l}$, $(\mathbf{J_l} = \mathbf{L} + \mathbf{S_l}, \mathbf{J} = \mathbf{J_l} + \bar{\mathbf{J}})$, where $\mathbf{J}$ denotes the total spin of baryon, and $\bar{\mathbf{J}}$ is the total spin of diquark, $\bar{\mathbf{J}} = \mathbf{S_d} + \mathbf{L_d}$).

Then, to estimate various terms and mixings of states, we use the transformations of bases (in what follows $\mathbf{S} = \mathbf{S_l} + \bar{\mathbf{J}}$)

$$|J; J_l\rangle = \sum_S (-1)^{(\bar{J}+S_l+L+J)} \sqrt{(2S+1)(2J_l+1)} \left\{ \begin{array}{ccc} \bar{J} & S_l & S \\ L & J & J_l \end{array} \right\} |J; S\rangle \tag{1.28}$$

and

$$|J; J_l\rangle = \sum_{J_d} (-1)^{(\bar{J}+S_l+L+J)} \sqrt{(2J_d+1)(2J_l+1)} \left\{ \begin{array}{ccc} \bar{J} & L & J_d \\ S_l & J & J_l \end{array} \right\} |J; J_d\rangle. \tag{1.29}$$

Thus, we have defined the procedure of calculations for the mass spectra of doubly heavy baryons. This procedure leads to results presented in the next section.

1.2. Numerical results

In this Section we present the results on the mass spectra with account for the spin-dependent splitting of levels. As we have clarified in the Introduction, the doubly heavy baryons with identical heavy quarks allow quite a reliable interpretation in terms of diquark quantum numbers (the summed spin and the orbital momentum). Dealing with the excitations of bc-diquark, we show the results on the spin-dependent splitting of the ground 1S-state, since the emission of soft gluon breaks the simple classification of levels for the higher excitations of such diquark.

For the doubly heavy baryons, the quark-diquark model of bound states obviously leads to the most reliable results for the system with the larger mass of heavy quark, i.e. for Ξ_{bb}.

For the quantum numbers of levels, we use the notations $n_d L_d n_l l_l$, i.e. we show the value of principal quantum number of diquark, its orbital momentum by a capital letter and the principal quantum number for the excitations of light quark and its orbital momentum by a lower-case letter. The splitting of Ξ_{bb} baryon levels was in detail considered in [26]. The states with the total spin $J = \frac{3}{2}$ (or $\frac{1}{2}$), can have different values of J_l, and, hence, they have a nonzero mixing, when we perform the calculations in the perturbation theory built over the states with the definite total momentum J_l of the light constituent quark. For $J = \frac{3}{2}$ the mixing matrix can be approximated by the diaginal matrix with a high accuracy. For $J = \frac{1}{2}$ the mixing of states with the different values of total spin-orbital momentum of light quark is strong. The analysis for the $1S2p$ and $2S2p$ levels in the system Ξ_{bb} was done in ref. [26], where one can see that the difference between the wave functions because of the slow change of diquark subsystem mass is not essential within the accuracy of the method.

The splittings of D and G levels in the diquark are less than 11 MeV, so that these corrections are small for the diquark excitations having the sizes less than the distance to the light quark, i.e. for the states with low value of principal quantum number, under the systematic uncertainty about $\delta M \approx 30 - 40$ MeV.

For the hyper-fine spin-spin splitting in the system of quark-diquark, we have

$$\Delta^{(l)}_{h.f.} = \frac{2}{9}\Big[J(J+1) - \bar{J}(\bar{J}+1) - \frac{3}{4}\Big]\alpha_s(2\mu T)\frac{1}{m_b m_l}|R_l(0)|^2, \tag{1.30}$$

where $R_l(0)$ is the radial wave function at the origin for the light constituent quark, and for the analogous shift of diquark level, we find

$$\Delta^{(d)}_{h.f.} = \frac{1}{9}\alpha_s(2\mu T)\frac{1}{m_b^2}|R_d(0)|^2. \tag{1.31}$$

The mass spectrum of Ξ^+_{bb} and Ξ^0_{bb} baryons is shown in Fig. 1.5, wherein we restrict ourselves by the presentation of S-, P- and D-wave levels, while the table containing the numerical values of masses for the Ξ_{bb} baryons is presented in [26].

We can see in Fig. 1.5 that the most reliable predictions are the masses of baryons $1S1s$ ($J^P = 3/2^+,\ 1/2^+$), $2P1s$ ($J^P = 3/2^-,\ 1/2^-$) and $3D1s$ ($J^P = 7/2^+, \dots 1/2^+$). The $2P1s$-level is quasi-stable, because the transition into the ground state requires the instantaneous change of both the orbital momentum and the summed spin of quarks inside the diquark. The analogous kind of transitions seems to be the transition between the states of ortho- and para-hydrogen in the molecule of H_2. This transition take place in a non-homogeneous external field due to the magnetic moments of other molecules. For the transition of $2P1s \to 1S1s$, the role of such the external field

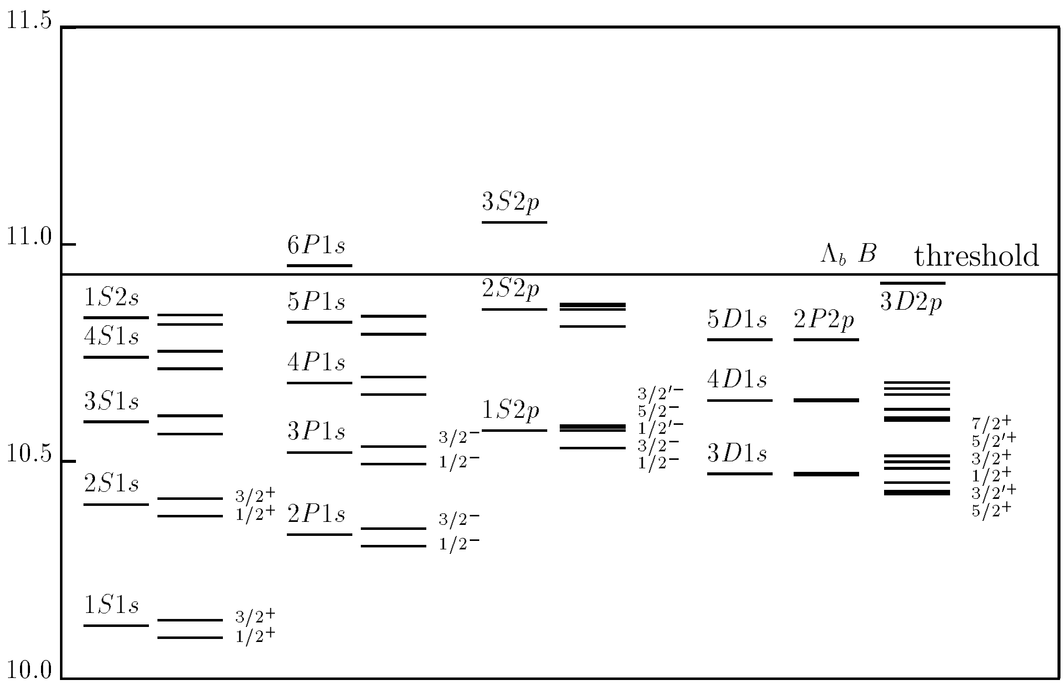

Figure 1.5. The spectrum of baryons, containing two b-quarks: Ξ_{bb}^- and Ξ_{bb}^0, with account for the spin-dependent splittings of low-lying excitations. The masses are given in GeV.

is played by the non-homogeneous chromo-magnetic field of the light quark. The corresponding perturbation has the form

$$\begin{aligned}\delta V &\sim \frac{1}{m_Q}[\mathbf{S}_1\cdot\mathbf{H}_1+\mathbf{S}_2\cdot\mathbf{H}_2-(\mathbf{S}_1+\mathbf{S}_2)\cdot\langle\mathbf{H}\rangle]\\ &= \frac{1}{2m_Q}(\nabla\cdot\mathbf{r_d})\,(\mathbf{S}_1-\mathbf{S}_2)\cdot\mathbf{H}\sim\frac{1}{m_Q}\frac{\mathbf{r_l}\cdot\mathbf{r_d}}{m_q r_l^5}\,(\mathbf{S}_1-\mathbf{S}_2)\cdot\mathbf{J_l}\,f(r_l),\end{aligned}$$

where $f(r_l)$ is a dimensionless nonperturbative function depending on the distance between of the light quark and diquark. The δV operator changes the orbital momentum of light quark, too. It results in the mixing between the states with the same values of J^P. If the splitting is not small (for instance, $2P1s-1S2p$, where $\Delta E\sim\Lambda_{QCD}$), then the mixing is suppressed as $\delta V/\Delta E\sim\frac{1}{m_Q m_q}\frac{r_d}{r_l^4}\frac{1}{\Delta E}\ll 1$. Since the admixture of $1S2p$ in the $2P1s$-state is low, the $2P1s$-levels are quasi-stable, i.e. their hadronic transitions into the ground state with the emission of π-mesons are suppressed as we have derived, though an additional suppression is given by a small value of phase space. Therefore, we have to expect the presence of narrow resonances in the mass spectra of pairs $\Xi_{bb}\pi$, as they are produced in the decays of quasi-stable states with $J^P=3/2^-,\ 1/2^-$. The experimental observation of such levels could straightforwardly confirm the existence of diquark excitations and provide the information on the character of dependency in $f(r_l)$, i.e. on the non-homogeneous chromo-magnetic field in the nonperturbative region.

Sure, the $3D1s$ $J^P=7/2^+,\ 5/2^+$ states are also quasi-stable, since in the framework of multipole expansion in QCD they transform into the ground state due to the quadrupole emission of gluon (the E2-transition with the hadronization $gq\to q'\pi$).

As for the higher excitations, the $3P1s$-states are close to the $1S2p$-levels with $J^P = 3/2^-,\ 1/2^-$, so that the operators changing both the orbital momentum of diquark and its spin, can lead to the essential mixing with an amplitude $\delta V_{nn'}/\Delta E_{nn'} \sim 1$, despite of suppression by the inverse heavy quark mass and small size of diquark. We are sure that the mixing slightly shifts the masses of states. The most important effect is a large admixture of $1S2p$ in $3P1s$. It makes the state to be unstable because of the transition into the ground $1S1s$-state with the emission of gluon (the E1-transition). This transition leads to decays with the emission of π-mesons[5].

The level $1S2p$ $J^P = 5/2^-$ has the definite quantum numbers of diquark and light quark motion, because there are no levels with the same values of J^P in its vicinity. However, its width of transition into the ground state and π-meson is not suppressed and seems to be large, $\Gamma \sim 100$ MeV.

For the transitions we have

$$\frac{3}{2}^- \to \frac{3}{2}^+ \pi \ \text{ in S}-\text{wave}, \quad \frac{3}{2}^- \to \frac{1}{2}^+ \pi \ \text{ in D}-\text{wave},$$

$$\frac{1}{2}^- \to \frac{3}{2}^+ \pi \ \text{ in D}-\text{wave}, \quad \frac{1}{2}^- \to \frac{1}{2}^+ \pi \ \text{ in S}-\text{wave}.$$

The D-wave transitions are suppressed by the ratio of low recoil momentum to the mass of baryon.

The width of state $J^P = 3/2^+$ is completely determined by the radiative electromagnetic M1-transition into the ground $J^P = 1/2^+$ state.

The calculation procedure described above leads to the results for the doubly charmed baryons as presented in Table 1.7.

Table 1.7. The mass spectrum of Ξ_{cc}^{++} and Ξ_{cc}^{+} baryons.

$(n_d L_d n_l L_l)$, J^P	mass, GeV	$(n_d L_d n_l L_l)$, J^P	mass, GeV
(1S 1s)$1/2^+$	3.478	(3P 1s)$1/2^-$	3.972
(1S 1s)$3/2^+$	3.61	(3D 1s)$3/2'^+$	4.007
(2P 1s)$1/2^-$	3.702	(1S 2p)$3/2'^-$	4.034
(3D 1s)$5/2^+$	3.781	(1S 2p)$3/2^-$	4.039
(2S 1s)$1/2^+$	3.812	(1S 2p)$5/2^-$	4.047
(3D 1s)$3/2^+$	3.83	(3D 1s)$5/2'^+$	4.05
(2P 1s)$3/2^-$	3.834	(1S 2p)$1/2'^-$	4.052
(3D 1s)$1/2^+$	3.875	(3S 1s)$1/2^+$	4.072
(1S 2p)$1/2^-$	3.927	(3D 1s)$7/2^+$	4.089
(2S 1s)$3/2^+$	3.944	(3P 1s)$3/2^-$	4.104

As we have already mentioned, the heavy diquark composed of the quarks of different flavors, turns out to be unstable under the emission of soft gluons. So, in the Fock state of doubly heavy baryon, there is a sizable nonperturbative admixture of configurations including the gluons and diquark with the various values of its spin S_d and orbital momentum L_d

$$|B_{bcq}\rangle = O_B |bc_{\bar{3}_c}^{S_d,L_d}, q\rangle + H_1 |bc_{\bar{3}_c}^{S_d\pm1,L_d}, g, q\rangle + H_2 |bc_{\bar{3}_c}^{S_d,L_d\pm1}, g, q\rangle + \dots,$$

[5]Remember, that the $\Xi_{QQ'}$-baryons are the iso-dublets.

whereas the amplitudes of H_1, H_2 are not suppressed with respect to O_B. In the heavy quarkonium, the analogous operators for the octet-color states are suppressed by the probability of emission by the nonrelativistic quarks inside a small volume determined by the size of singlet-color system of heavy quark and anti-quark. In the baryonic system under consideration, a soft gluon is restricted only by the ordinary scale of confinement, and, hence, there is no suppression.

We suppose that the calculations of masses for the excited Ξ_{bc} baryons are not so justified in the given scheme. Therefore, we present only the result for the ground state with $J^P = 1/2^+$

$$M_{\Xi'_{bc}} = 6.85 \text{ GeV}, \quad M_{\Xi_{bc}} = 6.82 \text{ GeV},$$

whereas for the vector diquark we have assumed that the spin-dependent splitting due to the interaction with the light quark is determined by the standard contact coupling of magnetic moments for the point-like systems. The picture for the baryon levels with no account for the spin-dependent perturbations suppressed by the heavy quark masses is shown in [26].

1.2.1. The doubly heavy baryons with the strangeness $\Omega_{QQ'}$

In the leading approximation, we suppose that the wave functions and the excitation energies of strange quark in the field of doubly heavy diquark repeat the characteristics for the analogous baryons containing the ordinary quarks u, d. Therefore, the level system of baryons $\Omega_{QQ'}$ reproduces that of $\Xi_{QQ'}$ up to an additive shift of the masses by the value of current mass of strange quark, $m_s \approx M(D_s) - M(D) \approx M(B_s) - M(B) \approx 0.1$ GeV.

Further, we suppose that the spin-spin splitting of $2P1s$ and $3D1s$ levels of $\Omega_{QQ'}$ is 20-30% less than in $\Xi_{QQ'}$ (the factor of $m_{u,d}/m_s$). As for the $1S2p$-level, the procedure described above can be applied. So, for Ω_{bb}, the matrix of mixing for the states with the different values of total momentum J_l practically can be assigned to be diagonal. This fact means that the following term of perturbation is dominant:

$$\frac{1}{4}\left(\frac{2\mathbf{L}\cdot\mathbf{S}_\text{l}}{2m_l^2}\right)\left(-\frac{dV(r)}{rdr}+\frac{8}{3}\alpha_s\frac{1}{r^3}\right).$$

Therefore, we can think that the splitting of $1S2p$ is determined by the factor of $m_{u,d}^2/m_s^2$ with respect to the splitting of corresponding Ξ_{bb}, i.e. it is 40% less than in Ξ_{bb}. Hence, the splitting is very small.

For the baryon Ω, the factor of m_s/m_c is not small. Hence, for $1S2p$, the mixing matrix is not diagonal, so that the arrangement of $1S2p$ states of Ω can be slightly different from that of Ξ.

The following peculiarity of $\Omega_{QQ'}$ is of great interest: the low-lying S- and P-excitations of diquark are stable. Indeed, even after taking into account the mixing of levels, a gluon emission makes a hadronization into the K-meson (the transitions of $\Omega_{QQ'} \to \Xi_{QQ'} + K$), while a single emission of π-meson is forbidden because of the conservation of iso-spin and strangeness. The hadronic transitions with kaons are forbidden because of insufficient splitting between the masses of $\Omega_{QQ'}$ and $\Xi_{QQ'}$. The decays with the emission of pion pairs belonging to the iso-singlet state, are suppressed by a small phase space or even forbidden. Thus, the radiative electromagnetic transitions into the ground state are the dominant modes of decays for the low-lying excitations of $\Omega_{QQ'}$.

1.2.2. Ω_{bbc} baryons

In the framework of quark-diquark picture, we can build the model for the baryons containing three heavy quarks, *bbc*. However, as we estimate, the size of diquark turns out to be comparable with the average distance to the charmed quark. So, the model assumption on the compact heavy diquark cannot be quite accurate for the calculations of mass levels in this case. The spin-dependent forces are negligibly small inside the diquark, as we have already pointed out above. The spin-spin splitting of vector diquark interacting with the charmed quark, is given by $\Delta(1s) = 33$ MeV, $\Delta(2s) = 18$ MeV. For $1S2p$, the level shifts are small. So, for the state $J^P = 1/2^-$ we have to add the correction of -33 MeV. For the $3D1s$-state the splitting is determined by the spin-spin interaction. The characteristics of excitations for the charmed quark in the model with the potential by Buchmüller and Tye have been presented above. Finally, we obtain the picture of Ω_{bbc} levels presented in Table 1.8.

Table 1.8. The mass spectrum of Ω^0_{bbc} baryons.

$(n_d L_d n_l L_l)$, J^P	mass, GeV	$(n_d L_d n_l L_l)$, J^P	mass, GeV
(1S 1s)$1/2^+$	11.12	(3D 1s)$3/2'^+$	11.52
(1S 1s)$3/2^+$	11.18	(3D 1s)$5/2'^+$	11.54
(2P 1s)$1/2^-$	11.33	(1S 2p)$1/2^-$	11.55
(2P 1s)$3/2^-$	11.39	(3D 1s)$7/2^+$	11.56
(2S 1s)$1/2^+$	11.40	(1S 2p)$3/2'^-$	11.58
(3D 1s)$5/2^+$	11.42	(1S 2p)$3/2^-$	11.58
(3D 1s)$3/2^+$	11.44	(1S 2p)$1/2'^-$	11.59
(3D 1s)$1/2^+$	11.46	(1S 2p)$5/2^-$	11.59
(2S 1s)$3/2^+$	11.46	(3P 1s)$3/2^-$	11.59
(3P 1s)$1/2^-$	11.52	(3S 1s)$1/2^+$	11.62

Further, the excitations of ground Ω^0_{bbc} state can strongly mix with large amplitudes because of small splittings between the levels, but they have small shifts of masses. This effect takes place for $3P1s - 1S2p$ with $J^P = 1/2^-$, $3/2^-$, and for $2S1s - 3D1s$ with $J^P = 1/2^+$, $3/2^+$. We suppose the prediction to be quite reliable for the states of $1S1s$ with $J^P = 1/2^+$, $3/2^+$, $1S2p$ with $J^P = 5/2^-$ and $3D1s$ with $J^P = 5/2^+$, $7/2^+$. For these excitations, we might definitely predict the widths of their radiative electromagnetic transitions into the ground state in the framework of multipole expansion in QCD. The widths for the transitions will be essentially determined by the amplitudes of admixtures, which have a strong model dependence. Therefore, the experimental study of electromagnetic transitions in the family of Ω^0_{bbc} baryons could provide a significant information on the mechanism of mixing between the different levels in the baryonic systems. The electromagnetic transitions combined with the emission of pion pairs, if not forbidden by the phase space, saturate the total widths of excited Ω^0_{bbc} levels. The characteristic value of total width is about $\Gamma \sim 10-100$ keV, in the order of magnitude.

Thus, the system of Ω^0_{bbc} can be characterized by a large number of narrow quasi-stable states.

1.3. Discussion

In this paper we have calculated the spectroscopic characteristics of baryons containing two heavy quarks, in the model with the quark-diquark factorization of wave functions. We have explored the nonrelativistic model of constituent quarks with the potential by Buchmüller and Tye. The region of applicability of such the approximations has been pointed out.

We have taken into account the spin-dependent relativistic corrections to the potential in the subsystems of diquark and light quark-diquark. Below the threshold of decay into the heavy baryon and heavy meson, we have found the system of excited bound states, which are quasi-stable under the hadronic transitions into the ground state. We have considered the physical reasons for the quasi-stability taking place for the baryons with two identical quarks. In accordance with the Pauli principle, the operators responsible for the hadronic decays and the mixing between the levels, are suppressed by the inverse heavy quark mass and the small size of diquark. This suppression is caused by the necessity of instantaneous change in both the spin and the orbital momentum of compact diquark. In the baryonic systems with two heavy quarks and the strange quark, the quasi-stability of diquark excitations is provided by the absense of transitions with the emission of both a single kaon and a single pion. These transitions are forbidden because of small splitting between the levels and the conservation of iso-spin and strangeness.

The characteristics of wave functions can be used in calculations of cross sections for the doubly heavy baryons in the framework of quark-diquark approximation.

The quark-diquark factorization in calculating the masses of ground states for the baryon systems with two heavy quarks was also considered in ref. [29], where the quasi-potential approach [30] was explored. There is a numerical difference in the choice of heavy quark masses, that leads to that in [29] the mass of doubly charmed diquark, say, about 100 MeV greater that the mass used in the above calculations. This difference determines the discrepacy of estimates for the masses of ground states presented in this paper and in [29]. We believe that this deviation between the quark masses is caused by the use of Cornell potential with the constant value of effective coulomb exchange coupling in contrast to the above consideration with the running coupling constant, that cancels the uncertainty in the arbitrary additive shift of energy. Furthermore, in the potential approach the masses of heavy quarks depend on the mentioned additive shift, which adjusted in the phenomemological models by comparing, say, the leptonic constants of heavy quarkonium calculated in the model with the values known from experiments. In the QCD motivated potential such the ambiguity of potential because of the additive shift is absent, so that the estimates of heavy quark masses have less uncertainities. Let us stress that in the Cornell model the leptonic constants were calculated by taking into account the one-loop corrections caused by the hard gluons. This correction is quite essential, in part, for the charmed quarks. The two-loop corrections are also important for the consideration of leptonic constants in the potential approach [24]. Moreover, in [29] the constituent mass of light quark is posed with no correlation with the normalization of potential, while we put the constituent mass to be a part of nonperturbative energy in the potential. This can lead to an additional deviation between the estimates of baryon masses about 50 MeV. Taking into account the above notes on the systematic differences, we can claim that the estimates of ground states masses for the baryons with two heavy quarks in [29] agree with the values obtained in the presented approach (see Table 1.9).

In ref. [31], following [29] in the framework of quasi-potential approach, the analysis of spin-dependent relativistic corrections was performed so that the overestimated, to our opinion, value of heavy diquark from [29] was used. Unfortunately, there is an evident mistake in the description of

Table 1.9. The masses of ground states M (in GeV) for the baryons with two heavy quarks calculated in various approaches (* denotes the results of authors in this review). The accuracy of predictions under the variation of model parameters is about 30-50 MeV. The systematic unceratinties are discussed in the text.

baryon	*	[31]	[29]	[32]	[33]	[34]	[35]
Ξ_{cc}	3.48	3.74	3.66	3.66	3.61	3.65	3.71
Ξ^*_{cc}	3.61	3.86	3.81	3.74	3.68	3.73	3.79
Ω_{cc}	3.59	3.76	3.76	3.74	3.71	3.75	3.89
Ω^*_{cc}	3.69	3.90	3.89	3.82	3.76	3.83	3.91
Ξ_{bb}	10.09	10.30	10.23	10.34	-	-	10.43
Ξ^*_{bb}	10.13	10.34	10.28	10.37	-	-	10.48
Ω_{bb}	10.18	10.34	10.32	10.37	-	-	10.59
Ω^*_{bb}	10.20	10.38	10.36	10.40	-	-	10.62
Ξ_{cb}	6.82	7.01	6.95	7.04	-	-	7.08
Ξ'_{cb}	6.85	7.07	7.00	6.99	-	-	7.10
Ξ^*_{cb}	6.90	7.10	7.02	7.06	-	-	7.13
Ω_{cb}	6.91	7.05	7.05	7.09	-	-	7.23
Ω'_{cb}	6.93	7.11	7.09	7.06	-	-	7.24
Ω^*_{cb}	6.99	7.13	7.11	7.12	-	-	7.27

calculations in [31], because both the parameter giving the relative contribution of scalar and vector part in the potential and the anomalous chromo-magnetic moment of heavy quark are denoted by the same symbol, that leads to numerical errors, since in [29] it was shown that these quantities have different values. This mistake enlarges the unceratinty about 100 MeV into th estimates of [31], so that we can consider that the results of [31] do not contradict with the presented description (see Table 1.9).

The estimates based on the hypothesis of pair interactions were presented in ref. [32], so that in the light of discussion given in the beggining of this chapter the difference about 200-300 MeV, that follows from values in Table 1.9, is not amazing. This deviation is, in general, related with the different character of interquark forces in the doubly heavy baryon, though the uncertainty in the heavy quark masses is also important.

In ref. [33] simple speculations based on the HQET with the heavy diquark were explored, so that the estimates depend on the supposed mass of diquark composed of two heavy quarks. In this way, if we neglected the binding energy in the diquark, that is evidently related with the choice of heavy quark masses, then we got the estimates of ground state masses shown in Table 1.9.

Finally, in [35] the analysis given in [36] was modified on the basis of interpolation formulae for the mass of ground state with account for the dependence of spin forces on both the wave functions and the effective coupling constant, which were changed under the quark contents of hadrons. In this way, the parameter of energy shift enters the fitting function, so that this parameter essentially changes under the transition from the description of mesons to baryons: $\delta_M \approx 80$ MeV $\longrightarrow \delta_B \approx 210$ MeV. This shift of energy provides a good agreement of fitting with the mass values for the mesons and baryons observed experimentally. However, if we suggest that the doubly heavy baryon is similar with the meson containing the local heavy source in the picture of strong interactions, then we should use the energy shift prescribed to the heavy mesons but the heavy baryons, wherein the

presense of system with two light quarks leads to the essential difference in the calculation of bound state masses, hence, to the energy shift different from the mesonic one. Such the substitution of parameters would lead to a more good agreement between the results of [35] (see Table 1.9) and the values obtained in this review.

Summarizing, we can claim that, first of all, in the framework of potential approach in the calculations of masses for the doubly heavy baryons the dominant uncertainty is caused by the choice of heavy quark masses, so that due to the adjustment on the systems with heavy quarks, the analysis presented in the QCD-motivated model of potential with the running coupling constant at short distances and the linear nonperturbative term confining quarks at large distances, gives the most reliable predictions.

A new field of interest for the investigations is radiative, electromagnetic or hadronic, transitions between the quasi-stable states in families of baryons with two heavy quarks. A first step in the study of this problem was recently done in [37], wherein some preliminary results were obtained on the electromagnetic transitions between the levels of Ξ_{bc}.

Chapter 2. Sum rules of nonrelativistic QCD: two-point correlators

In the framework of potential models in Chapter 1 we have described the families of baryons with two heavy quarks, which form a set of narrow excited states in addition to the ground states, so that the mass spectrum is very similar with the system of levels in the heavy quarkonium. In the method of QCD sum rules [7] for the two-point correlators of baryonic currents, the masses and coupling constants for the baryons with two heavy quarks were calculated in [38]. However, the analysis in [38] has some disadvantages connected to an unstable divergency of sum rules in the range of parameters determining the baryonic currents. This leads to quite large uncertainties in calculations.

In this chapter we investigate the sum rules of NRQCD for the two-point correlators of currents corresponding to the baryons with two heavy quarks. The main physical argument of such the consideration is the nonrelativistic motion of heavy quarks inside the diquark of small size, that interacts with the light quark. This fact leads to some definite expressions for the structure of baryonic currents written down in terms of nonrelativistic heavy quark fields. In the leading order of inverse heavy quark mass and relative velocity of heavy quarks inside the diquark, we have to take into account the hard gluon corrections in order to derive the relations between the correlators of nonrelativistic quarks in NRQCD and the correlators in full QCD. The corresponding anomalous dimensions for the baryonic currents were calculated up to two loops in ref. [39]. The structure of currents in NRQCD corresponds to some choice of parameters in expressions of full QCD. These values of parameters are posed in the range of significant uncertainty observed in the analysis performed in [38]. We find a simple physical reason for failure of stability in this case: the behaviour of quantities versus the scheme parameters of sum rules (the Borel variable or the number of spectral density moment) is determined by forming the doubly heavy diquark inside the baryon and, hence, the difference between the masses of baryon and diquark. This difference of masses determines the basic characteristics of correlators if we do not take into account the corrections connected to the nonperturbative interaction of doubly heavy diquark with the light quark in the baryon. In the NRQCD sum rules, the introduction of such the interaction is related with the nonperturbative condensates caused by operators of higher dimensions. We show that the better stability and lower uncertainty of sum rules can be achieved by taking into account the product of quark and gluon condensates in addition to quark, gluon and mixed condensates. Moreover, we accurately introduce the coulomb α_s/v-corrections inside the heavy diquark. These corrections enforce the relative contribution of perturbative part of sum rules in comparison with the contribution of condensates into the correlators under consideration.

Further, we comparatively analize the sum rules for the doubly heavy baryons with the both strange and light massless quarks.

In Section 2.1 we define the currents and represent the spectral densities in the NRQCD sum rules for various operators included into the consideration. Section 2.2 is devoted to numerical estimates. We calculate the masses of ground states, which are in a good agreement with the values obtained in potential models. Finally, we briefly summarize the results.

2.1. Sum rules for doubly heavy baryons

2.1.1. Baryonic currents

The currents for the baryons with two heavy quarks $\Xi^{\diamond}_{cc}$, $\Xi^{\diamond}_{bb}$ and $\Xi'^{\diamond}_{bc}$, where $\diamond$ denotes the electric charge of baryon depending on the flavor of light quark, corespond to the quantum numbers of spin and parity $j_d^P = 1^+$ and $j_d^P = 0^+$ for the systems of heavy diquark with the symmetric and anti-symmteric structure of flavor matrix, correspondingly (if the identical heavy quarks form the diquark then the scalar ground state $j_d^P = 0^+$ is forbidden). Adding the light quark to the system of heavy quarks gives $j^P = \frac{1}{2}^+$ for the $\Xi'^{\diamond}_{bc}$ baryons and the couple of degenerate states $j^P = \frac{1}{2}^+$ and $j^P = \frac{3}{2}^+$ for the baryons $\Xi^{\diamond}_{cc}$, $\Xi^{\diamond}_{bc}$, $\Xi^{\diamond}_{bb}$ and $\Xi^{*\diamond}_{cc}$, $\Xi^{*\diamond}_{bc}$, $\Xi^{*\diamond}_{bb}$. Generally, the structure of baryonic currents with two heavy quarks is written down in the form

$$J = [Q^{iT} C\Gamma\tau Q^j]\Gamma' q^k \varepsilon_{ijk}. \tag{2.1}$$

Here T denotes the transposition, C is the charge conjugation matrix with the properties $C\gamma_\mu^T C^{-1} = -\gamma_\mu$ and $C\gamma_5^T C^{-1} = \gamma_5$, i, j, k are color indices and τ is a matrix in the flavor space. The effective static field of the heavy quark is denoted by Q. To the leading order over both the relative velocity of heavy quarks and their inverse masses, this field contains the "large" component only in the hadron rest frame.

Here, unlike the case of baryons with a single heavy quark [40], there is the only independent current component J for each of the ground state baryon currents. They equal

$$\begin{aligned} J_{\Xi'^{\diamond}_{QQ'}} &= [Q^{iT} C\tau\gamma_5 Q^{j\prime}] q^k \varepsilon_{ijk}, \\ J_{\Xi^{\diamond}_{QQ}} &= [Q^{iT} C\tau\boldsymbol{\gamma}^m Q^j]\cdot \boldsymbol{\gamma}_m \gamma_5 q^k \varepsilon_{ijk}, \\ J^n_{\Xi^{*\diamond}_{QQ}} &= [Q^{iT} C\tau\boldsymbol{\gamma}^n Q^j] q^k \varepsilon_{ijk} + \frac{1}{3}\boldsymbol{\gamma}^n [Q^{iT} C\boldsymbol{\gamma}^m Q^j]\cdot \boldsymbol{\gamma}_m q^k \varepsilon_{ijk}, \end{aligned} \tag{2.2}$$

where $J^n_{\Xi^{*\diamond}_{QQ}}$ satisfies the spin-3/2 condition $\boldsymbol{\gamma}_n J^n_{\Xi^{*\diamond}_{QQ}} = 0$. The flavor matrix τ is anti-symmetric for $\Xi'^{\diamond}_{bc}$ and symmetric for $\Xi^{\diamond}_{QQ}$ and $\Xi^{*\diamond}_{QQ}$. The currents written down in Eq. (2.2) are taken in the rest frame of hadrons. The corresponding expressions in a general frame moving with a velocity v^μ can be obtained by the substitution of $\boldsymbol{\gamma}^m \to \gamma_\perp^\mu = \gamma^\mu - \not{v} v^\mu$.

Similar expressions can be written down for the doubly heavy baryons with the strange quark.

To compare with the full QCD analysis we represent the expression for the $J_{\Xi'^{\diamond}_{bc}}$ current given in [38]

$$J_{\Xi'^{\diamond}_{bc}} = \{r_1[u^{iT} C\gamma_5 c^j] b^k + r_2[u^{iT} C c^j]\gamma_5 b^k + r_3[u^{iT} C\gamma_5\gamma_m u c^j]\gamma^\mu b^k\}\varepsilon_{ijk}, \tag{2.3}$$

so that the NRQCD structure can be obtained by the choice of $r_1 = r_2 = 1$ and $r_3 = 0$ and the anti-symmetric permutation of c and b flavors. This connection can be achieved by the nonrelativistic limit of full QCD spinors of heavy quarks, so that in the leading order of $1/m_Q$-expansion the "large" components of spinors contribute only. Therefore, for the ground states of douby heavy baryons containing the heavy quarks with the identical flavors, the leading approximation of NRQCD leads to the only structure of baryonic current expressed in terms of nonrelativistic spinors of heavy quark, since the total spin of heavy diquark is fixed by $S = 1$ because of the Pauli principle. The corrections of the first $1/m_Q$-order can contribute with the other Lorentz structures, of course. However, we deal with the leading approximation of NRQCD in the present paper. For the ground states of doubly heavy baryons containing the heavy quarks with the different flavors, the 1/2-spin state of baryon can contain the mixture of diquark states with $S = 1$ and $S = 0$, as it does in

full QCD. We perform the separate consideration of these two currents in NRQCD, and such the approach can generally be not optimal in full QCD currents. Nevertheless, as we show, in the leading order of NRQCD there are relations between the masses and coupling constants of baryons because of the spin symmetry, so that the NRQCD does not distinguish these spin states untill the spin dependent $1/m_Q$-corrections are taken into account. In addition, as we have already mentioned, the analysis in the sum rules of full QCD was done with a large uncertainty beacuse of difference in the evaluations of coupling constants from two correlation functions [38], and the authors noted that this uncertainty became large in a region of parameters $r_{1,2}$ defined above, so that this region of bad accuracy is placed in the vicinity of point gining the NRQCD choice in (2.3). To get rid this disadvantage and to find its reason, we analyze the NRQCD sum rules in details.

2.1.2. Description of the method

In this section we describe steps required for the evaluation of two-point correlation functions in the NRQCD approximation and the connection to physical characteristics of doubly heavy baryons. We start from the correlator of two baryonic currents with the half spin

$$\Pi(w) = i\int d^4x e^{ipx}\langle 0|TJ(x), \bar{J}(0)|0\rangle = \not{v}F_1(w) + F_2(w), \tag{2.4}$$

where w is defined by $p^2 = (\mathcal{M} + w)^2$, while $\mathcal{M} = m_Q + m_{Q'} + m_s$, $m_{Q,Q'}$ are the heavy quark masses, and m_s is the strange quark mass. Sure, the correlators for the baryonic currents with the light quarks instead of the strange one can be obtained, if we put $m_s \to m_{u,d} \approx 0$ in expressions below. The appropriate definitions of scalar formfactors for the 3/2-spin baryon are given by the following:

$$\Pi_{\mu\nu}(w) = i\int d^4x e^{ipx}\langle 0|T\{J_\mu(x), \bar{J}_\nu(0)\}|0\rangle = -g_{\mu\nu}[\not{v}\tilde{F}_1(w) + \tilde{F}_2(w)] + \dots, \tag{2.5}$$

where we do not concern for distinct Lorentz structures. The scalar correlators F can be evaluated in a deep euclidean region by employing the Operator Product Expansion (OPE) for the chronological product of baryonic currents in the framework of NRQCD, for instance, in Eqs. (2.4), (2.5),

$$F_{1,2}(w) = \sum_d C_d^{(1,2)}(w) O_d, \tag{2.6}$$

where O_d denotes the local operator with a given dimension d: $O_0 = \hat{1}$, $O_3 = \langle\bar{q}q\rangle$, $O_4 = \langle\frac{\alpha_s}{\pi}G^2\rangle$, $O_{d>4} = \dots$, and the functions $C_d(w)$ are the corresponding Wilson coefficients of OPE.

In this review we take into accouont the nonperturbative terms connected to the quark and gluon condensates, their product and mixed condensates. For the contribution of quark condensate operator we explore the following OPE for the correlator of two quark fields: [41]:

$$\langle 0|T\{q_i^a(x)\bar{q}_j^b(0)|0\rangle \;=\; -\frac{1}{12}\delta^{ab}\delta_{ij}\langle\bar{q}q\rangle\cdot[1 + \frac{m_0^2x^2}{16} + \frac{\pi^2x^4}{288}\langle\frac{\alpha_s}{\pi}G^2\rangle + ...], \tag{2.7}$$

where the value of mixed condensate is parametrized by introducing the variable m_0^2, which is numerically determined as $m_0^2 \approx 0.8$ GeV^2. Taking into account the nonzero mass of strange quark

in the framework of OPE we get the following expression for the quark condensate up to terms of fourth order in x [42]:

$$\begin{aligned}\langle 0|Ts_i^a(x)\bar{s}_j^b(0)|0\rangle = & -\frac{1}{12}\delta^{ab}\delta_{ij}\langle\bar{s}s\rangle\cdot\left[1+\frac{x^2(m_0^2-2m_s^2)}{16}+\frac{x^4(\pi^2\langle\frac{\alpha_s}{\pi}G^2\rangle-\frac{3}{2}m_s^2(m_0^2-m_s^2))}{288}\right] \\ & + im_s\delta^{ab}x_\mu\gamma_{ij}^\mu\cdot\langle\bar{s}s\rangle\cdot\left[\frac{1}{48}+\frac{x^2}{24^2}\left(\frac{3m_0^2}{4}-m_s^2\right)\right] = \\ & -\delta^{ab}\langle\bar{s}s\rangle\cdot(\mathcal{P}_0\delta_{ij}+\mathcal{P}_1x_\alpha\gamma_{ij}^\alpha+\mathcal{P}_2\delta_{ij}x^2+\mathcal{P}_3x_\alpha\gamma_{ij}^\alpha x^2+\mathcal{P}_4\delta_{ij}x^4).\end{aligned} \tag{2.8}$$

Note that at $m_s\neq 0$ the expansion of quark condensate (2.8) gives contributions in both correlators in contrast with the sum rules for $\Xi_{QQ'}$ [43], where putting $m_s=0$ and neglecting the higher condensates, the authors found the factorization of diquark correlator in F_2 and full baryonic correlator in F_1. This factorization led to the systematic instability for the estimates in the sum rules.

We write down the Wilson coefficients in front of unity and quark-gluon operators by making use of the dispersion relation over w,

$$C_d(w)=\frac{1}{\pi}\int_0^\infty\frac{\rho_d(\omega)d\omega}{\omega-w}, \tag{2.9}$$

where ρ denotes the imaginary part of corresponding Wilson coefficient in the physical region of NRQCD. Thus, the calculation of Wilson coefficients for the operators under consideration is reduced to the problem on the derivation of corresponding spectral densities.

To relate the NRQCD correlators to the real hadrons, we use the dispersion representation for the two-point function with the physical spectral density given by the appropriate resonance and continuum part. The coupling constants of baryons are defined by the following expressions:

$$\begin{aligned}\langle 0|J(x)|\Xi(\Omega)_{QQ}^\diamond(p)\rangle &= iZ_{\Xi(\Omega)_{QQ}^\diamond}u(v,M_{\Xi(\Omega)})e^{ipx}, \\ \langle 0|J^m(x)|\Xi(\Omega)_{QQ}^{*\diamond}(p,\lambda)\rangle &= iZ_{\Xi(\Omega)_{QQ}^{*\diamond}}u^m(v,M_{\Xi(\Omega)})e^{ipx},\end{aligned}$$

where the spinor field with the four-velocity v and mass M satisfies the equation

$$\not{v}u(v,M)=u(v,M),$$

and $u^m(v,M)$ denotes the transversal spinor, so that $(\gamma^m-v^m\not{v})u^m(v,M)=0$.

We suppose that the continuum densities starting from the threshold ω_{cont}, is modelled by the NRQCD expressions. Then, in the sum rules equalizing the correlators in NRQCD and those of given by the physical states, we assume the model of continuum given by the calculated perturbative term. This model cannot be exact because of binding effects as well as the truncation of perturbative expansion in the given order of α_s. Therefore, the integration above ω_{cont} cannot be strictly cancelled, and the model introduces the implicit dependence of masses and couplings on the choice of value ω_{cont}. This dependece causes an uncertainty, which is not essential in comparison with uncertainties following from another methodics and the variation of quark masses.

Then we use the nonrelativistic expressions for the physical spectral functions

$$\rho_{1,2}^{phys}(\omega)=\frac{M}{2\mathcal{M}}|Z|^2\delta(\bar{\Lambda}-\omega), \tag{2.10}$$

where we have performed the substitution $\delta(p^2 - M^2) \to \frac{1}{2\mathcal{M}}\delta(\bar{\Lambda} - w)$, here $\bar{\Lambda}$ is the binding energy of baryon and $M = \mathcal{M} + \bar{\Lambda}$. The nonrelativistic dispersion relation for the hadronic part of sum rules has the form

$$\int \frac{\rho_{1,2}^{phys} d\omega}{\omega - w} = \frac{1}{2\mathcal{M}} \frac{|Z|^2}{\bar{\Lambda} - w}. \tag{2.11}$$

Further, we write down the correlators in the deep underthreshold point of $w = -\mathcal{M} + t$ at $t \to 0$, which corresponds to the limit of $p^2 \to 0$. The approximation of hadronic part by the only bound state leads to the expression, which can be expanded in series of t. Thus, the sum rules result in the equality of coefficients at the same powers of t

$$\frac{1}{\pi} \int_0^{\omega_{cont}} \frac{\rho_{1,2}\, d\omega}{(\omega + \mathcal{M})^n} = \frac{M}{2\mathcal{M}} \frac{|Z|^2}{M^n}, \tag{2.12}$$

where ρ_j contains the contributions given by various operators in OPE for the corresponding scalar functions F_j. Introducing the following notation for the n-th moment of two-point correlation function:

$$\mathcal{M}_n = \frac{1}{\pi} \int_0^{\omega_{cont}} \frac{\rho(\omega)\, d\omega}{(\omega + \mathcal{M})^{n+1}}, \tag{2.13}$$

for the baryon mass M_Ξ we have the following estimate:

$$M[n] = \frac{\mathcal{M}_n}{\mathcal{M}_{n+1}}, \tag{2.14}$$

and the coupling constants are determined by the expression

$$|Z[n]|^2 = \frac{2\mathcal{M}}{M} \mathcal{M}_n M^{n+1}, \tag{2.15}$$

where we see the dependence of sum rule results on the scheme parameter. Therefore, we have to find the region of parameters, wherein, first, the results are stable with respect to variation of n, and, second, the both correlation functions F_1 and F_2 repeats the same values of physical quantities: the masses and coupling constants. The problem of analysis given in the full QCD was the existance of significant difference between the masses and coupling constants calculated under different F.

2.1.3. Calculating the spectral densities

In this subsection we present analytical expression for the perturbative spectral functions in the NRQCD approximation. The evaluation of spectral densities involves the standard use of Cutkosky rules [44] with some modifications motivated by NRQCD. We use the rule under which the jump of two-point function under study is calculated by the following substitutions for the propogators of heavy and light quarks, respectively:

$$\text{heavy quark :} \qquad \frac{1}{p_0 - (m + \frac{\mathbf{p}^2}{2m})} \to 2\pi i \cdot \delta(p_0 - (m + \frac{\mathbf{p}^2}{2m})),$$

$$\text{light quark :} \qquad \frac{1}{p^2 - m^2} \to 2\pi i \cdot \delta(p^2 - m^2).$$

We derive the spin symmetry relations for all the spectral densities due to the fact that in the leading order of the heavy quark effective theory the spins of heavy quarks are decoupled, so

$$\rho_{1,\Omega(\Xi)^{\diamond}_{QQ}} = 3\rho_{1,\Omega(\Xi)'^{\diamond}_{QQ'}} = 3\rho_{1,\Omega(\Xi)^{*\diamond}_{QQ}}, \tag{2.16}$$

$$\rho_{2,\Omega(\Xi)^{\diamond}_{QQ}} = 3\rho_{2,\Omega(\Xi)'^{\diamond}_{QQ'}} = 3\rho_{2,\Omega(\Xi)^{*\diamond}_{QQ}}, \tag{2.17}$$

and we have the following relation for the baryon couplings in NRQCD:

$$|Z_{\Omega(\Xi)}|^2 = 3|Z_{\Omega'(\Xi)}|^2 = 3|Z_{\Omega(\Xi)^*}|^2. \tag{2.18}$$

For the perturbative spectral densities $\rho_{1,H}(\omega)$ and $\rho_{2,H}(\omega)$ in front of unit operator for F_1 and F_2, respectively, we explore the smallness of strange quark current mass with respect to the masses of heavy quarks and make the expansion in series of m_s. So, we get the following expressions:

$$\rho_{1,\Omega'^{\diamond}_{QQ'}}(\omega) = \frac{\sqrt{2}(m_{QQ'}\omega)^{3/2}}{15015\pi^3(\mathcal{M}_{diq}+\omega)^3}(\eta_{1,0}(\omega) + m_s\eta_{1,1}(\omega) + m_s^2\eta_{1,2}(\omega)), \tag{2.19}$$

so that $m_{QQ'} = m_Q m_{Q'}/(m_Q + m_{Q'})$ is the reduced diquark mass,, $\mathcal{M}_{diq} = m_Q + m_{Q'}$, and the coefficients of spectral densities η are presented in Appendix I[6]. The first term of this expansion reproduces the result obtained in [43] for the zero mass of light quark. For the strange baryons the perturbative density $\rho_{2,\Omega'^{\diamond}_{QQ'}}$ is prportional to m_s, and it is not equal to zero

$$\rho_{2,\Omega'^{\diamond}_{QQ'}}(\omega) = \frac{2\sqrt{2}\omega(m_{QQ'}\omega)^{3/2}m_s}{105\pi^3(\mathcal{M}_{diq}+\omega)^2}(\eta_{2,0} + m_s\eta_{2,1} + m_s^2\eta_{2,2}). \tag{2.20}$$

In the leading order of perturbative NRQCD the correlators F_2 are equal to zero for the massless light quark. This fact is caused by the absense of interaction between the light quark and the heavy diquark in this order, and, therefore, there is no massive term in this correlator.

The coulomb interaction inside the diquark can be taken into account by the introduction of Sommerfeld factor $\mathbf{C}$ for the spectral density of diquark before the integration over the invariant mass of diquark in order to get the baryonic spectral densities, so that

$$\rho^{\mathbf{C}}_{diquark} = \rho^{bare}_{diquark}\cdot\mathbf{C} \tag{2.21}$$

whereas

$$\mathbf{C} = \frac{2\pi\alpha_s}{3v_{QQ'}}\left[1 - \exp\left(-\frac{2\pi\alpha_s}{3v_{QQ'}}\right)\right]^{-1}, \tag{2.22}$$

where we have taken into account the anti-triplet color structure of diquark, and $v_{QQ'}$ denotes the relative velocity of heavy quarks inside the diquark:

$$v_{QQ'} = \sqrt{1 - \frac{4m_Q m_{Q'}}{Q^2 - (m_Q - m_{Q'})^2}}, \tag{2.23}$$

where Q^2 is the square of heavy diquark four-momentum. In NRQCD we take the limit of low velocities, so that denoting the diquark invariant mass squared by $Q^2 = (\mathcal{M}_{diq} + \epsilon)^2$, we find

$$\mathbf{C} = \frac{2\pi\alpha_s}{3v_{QQ'}}, \quad v^2_{QQ'} = \frac{\epsilon}{2m_{QQ'}},$$

at $\epsilon \ll m_{QQ'}$. The modified spectral densities are equal to

$$\rho_1^{\mathbf{C}}(\omega) = \frac{m^2_{QQ'}\alpha_s\omega(2\mathcal{M}_{diq}+\omega)}{6\pi^2(\mathcal{M}_{diq}+\omega)^3}(\eta^{\mathbf{C}}_{1,0} + m_s\eta^{\mathbf{C}}_{1,1} + m_s^2\eta^{\mathbf{C}}_{1,2}). \tag{2.24}$$

[6]In what follows the coefficients of spectral densities, which are not given explicitly, are also given in Appendix I.

The leading term gives the result for the zero mass of light quark [43]. For $\rho^{\mathbf{C}}_{2,\Omega'^{\diamond}_{QQ'}}$ we have

$$\rho_2^{\mathbf{C}} = \frac{m_s m_{QQ'}^2 (2\mathcal{M}_{diq}+\omega)\omega\alpha_s}{2\pi(\mathcal{M}_{diq}+\omega)^2}(\eta^{\mathbf{C}}_{2,0} + m_s\eta^{\mathbf{C}}_{2,1} + m_s^2\eta^{\mathbf{C}}_{2,2}). \tag{2.25}$$

The truncated expansion in the mass of light quark leads to a small deviation about 0.5% from the exact integral representation, so that this approximation is quite justified with the initial three terms in the explicit analytic form.

Further, the spectral functions, connected to the condensates of light quarks and gluons, can be derived in the analogous way. For the moments of coefficients in front of quark condensate terms we have the following expressions:

$$\begin{aligned} \mathcal{M}^{(1)}_{\bar{q}q}(n) &= -\frac{(n+1)!}{n!}\mathcal{P}_1\mathcal{M}^{diq}(n+1) + \frac{(n+3)!}{n!}\mathcal{P}_3\mathcal{M}^{diq}(n+3) \\ \mathcal{M}^{(2)}_{\bar{q}q}(n) &= \mathcal{P}_0\mathcal{M}^{diq}(n) - \frac{(n+2)!}{n!}\mathcal{P}_2\mathcal{M}^{diq}(n+2) + \frac{(n+4)!}{n!}\mathcal{P}_4\mathcal{M}^{diq}(n+4), \end{aligned} \tag{2.26}$$

where we have used the coefficients of expansion in x, $\mathcal{P}_i$, in Eq. (2.8), while the n-th moment of two-point correlation function for the diquark is denoted by $\mathcal{M}^{diq}(n)$, and it is derived under the integration of spectral density

$$\rho_{diq} = \frac{12\sqrt{2}m_{QQ'}^{3/2}\sqrt{\omega}}{\pi}, \tag{2.27}$$

which should be multiplied by the Sommerfeld factor $\mathbf{C}$, where the variable ϵ is substituted by ω, since in this case there is no integration over the invariant mass of diquark. The modified density is equal to

$$\rho^{\mathbf{C}}_{diq} = \frac{48\pi\alpha_s m_{QQ'}^2}{3}, \tag{2.28}$$

and it does not depend on ω.

It is interesting to stress that in NRQCD the light quark condensate contributes to the F_2 correlators, only. This fact has a simple physical explanation: to the leading order the light quark operator can be factorized in the expression for the correlator of baryonic currents. Indeed, we can write down for the condensate contribution

$$\langle 0|T\{J(x),\bar{J}(0)\}|0\rangle \Rightarrow \langle 0|T\{q_i^a(x)\bar{q}_i^a(0)|0\rangle \cdot \frac{\hat{1}}{12}\cdot\langle 0|T\{J_d^j(x),\bar{J}_d^j(0)\}|0\rangle + \dots,$$

where $J_d^j(x)$ denotes the appropriate diquark current with the color index j, as it is defined by the baryon structure in Eqs. (2.2). So, we see that the restriction by the first term independent of x in the expansion for the quark correlator in (2.7) results in the independent contribution of diquark correlator to the baryonic one. Then, since the diquark correlator is isolated in F_2 from the baryonic formfactor F_1, the NRQCD sum rules lead to the evaluation of diquark masses and couplings from F_2, and estimation of baryon masses and couplings from F_1. These masses and couplings are different. The positive point is the possibility to calculate the binding energy for the doubly heavy baryons $\bar{\Lambda} = M_\Xi - \mathcal{M}_{diq}$. The disadvantage is the instability of NRQCD sum rules at this stage, since the various formfactors or correlators lead to the different results. In sum rules of full QCD various choices of parameters in the definitions of baryonic currents result in an admixture of pure diquark correlator in various formfactors, so that the estimations acquire huge uncertainties.

Say, the characteristic ambiguity in the evaluation of baryon mass in full QCD is about 300 MeV, i.e. the value close to the expected estimate of $\bar{\Lambda}$. The analysis in the framework of NRQCD makes this result to be not unexpectable. Moreover, it is quite evident that the introduction of interactions between the light quark and the doubly heavy diquark destroys the factorization of diquark correlator. Indeed, we see that due to the higher terms in expansion (2.7), the diquark factorization is explicitly broken, which has to result in the convergency of estimates obtained from F_1 and F_2. Below we show numerically that this fact is valid. Technically, we point out that the contribution to the moments of spectral density, determined by the light quark condensate including the mixed condensate and the product of quark and gluon condensates, can be calculated after the exploration of (2.7), so that

$$\mathcal{M}_n^{q\bar{q}} = \mathcal{M}_n^{\langle\bar{q}q\rangle} - \frac{(n+2)!}{n!}\frac{m_0^2}{16}\mathcal{M}_{n+2}^{\langle\bar{q}q\rangle} + \frac{(n+4)!}{n!}\frac{\pi^2}{288}\langle\frac{\alpha_s}{\pi}G^2\rangle\mathcal{M}_{n+4}^{\langle\bar{q}q\rangle}. \tag{2.29}$$

For the corrections determined by the gluon condensate and written down for the operator $O_4 = \langle\frac{\alpha_s}{\pi}G^2\rangle$, we have

$$\rho_1^{G^2}(\omega) = \frac{(m_Q^2+m_{Q'}^2+11m_Qm_{Q'})m_{QQ'}^{5/2}\sqrt{\omega}}{21\cdot 2^{10}\sqrt{2}\pi m_Q^2m_{Q'}^2(\mathcal{M}_{diq}+\omega)^2}(\eta_{1,0}^{G^2}+m_s\eta_{1,1}^{G^2}+m_s^2\eta_{1,2}^{G^2}). \tag{2.30}$$

For the nonzero mass of light quark the density $\rho_2^{G^2}(\omega)$ is proportional to m_s and equal to

$$\rho_2^{G^2}(\omega) = \frac{m_s(m_Q^2+m_{Q'}^2+11m_Qm_{Q'})m_{QQ'}^{5/2}\sqrt{\omega}}{3\cdot 2^{9}\sqrt{2}\pi m_Q^2m_{Q'}^2(\mathcal{M}_{diq}+\omega)}(\eta_{2,0}^{G^2}+m_s\eta_{2,1}^{G^2}), \tag{2.31}$$

so that

$$\eta_{2,0}^{G^2} = -(9\mathcal{M}_{diq}+\omega), \quad \eta_{2,1}^{G^2} = \frac{9\mathcal{M}_{diq}+\omega}{\mathcal{M}_{diq}+\omega}. \tag{2.32}$$

For the product of condensates $\langle\bar{q}q\rangle\langle\frac{\alpha_s}{\pi}G^2\rangle$, wherein the gluon fields are connected to the heavy quarks in contrast to the light quark, we have computed the contribution to the two-point correlation function itself. It has the following form:

$$F_2^{\bar{q}qG^2}(\omega) = -\frac{m_{QQ'}^{5/2}(m_Q^2+m_{Q'}^2+11m_Qm_{Q'})}{2^9\sqrt{2}\pi m_Qm_{Q'}(-\omega)^{5/2}}, \tag{2.33}$$

and $F_1^{\bar{q}qG^2}(\omega)=0$, so that we restrict the consideration by the operators with the dimension not greater than 7, while the nonzero contribution to F_1 appears in the fifth order of expansion (2.8). The obtained result is presented in the form, which allows the analytic continuation over $\omega = -\mathcal{M}+w$.

Thus, we provide the NRQCD sum rules, where we take into account the perturbative terms and the vacuum expectations of quark-gluon operators up to the contributions by the light quark condensate, gluon condensate, their product and the mixed condensate. Note, that the product of condensates is essential for the doubly heavy baryons, and we present the full NRQCD expression for this term, including the interaction of nonperturbative gluons with both the light and heavy quarks. The correct introduction of coulomb-like interactions is done for the perturbative spectral densities of heavy diquark, which is important for the nonrelativistic heavy quarks. Finally, we find the spin-symmetry relation for the baryon couplings in NRQCD

$$|Z_\Xi|^2 = 3|Z_{\Xi'}|^2 = 3|Z_{\Xi^*}|^2.$$

2.1.4. Anomalous dimensions for the baryonic currents

To connect the NRQCD sum rules to the quantities in full QCD we have to take into account the anomalous dimensions of effective baryonic currents with the nonrelativistic quarks. They determine the factors, which have to multiply the NRQCD correlators to obtain the values in full QCD. Indeed, to the leading order of NRQCD we have the relation

$$J^{QCD} = \mathcal{K}_J(\alpha_s, \mu_{\rm soft}, \mu_{\rm hard}) \cdot J^{NRQCD},$$

where the coefficient $\mathcal{K}_J(\alpha_s, \mu_{\rm soft}, \mu_{\rm hard})$ depends on the normalization scale $\mu_{\rm soft}$ and obeys the matching condition at the starting point of $\mu_{\rm hard} = \mathcal{M}_{diq}$. The anomalous dimensions of NRQCD currents are independent of the diquark spin structure in the leading order. They are equal to [39]

$$\begin{aligned}
\gamma &= \frac{d\ln C_J(\alpha_s,\mu)}{d\ln(\mu)} = \sum_{m=1}^{\infty}\left(\frac{\alpha_s}{4\pi}\right)^m \gamma^{(m)}, \\
\gamma^{(1)} &= \Big(-2C_B(3a-3) + 3C_F(a-2)\Big), \\
\gamma^{(2)} &= \frac{1}{6}(-48(-2+6\zeta(2))C_B^2 + C_A((104-240\zeta(2))C_B - 101C_F) \\
&\quad -64C_B n_f T_F + C_F(-9C_F + 52 n_f T_F)),
\end{aligned} \tag{2.34}$$

where $C_F = (N_c^2-1)/2N_c$, $C_A = N_c$, $C_B = (N_c+1)/2N_c$, and $T_F = 1/2$ for $N_c = 3$, n_f being the number of light quarks. In Eq.(2.34) we give the one-loop result with the arbitrary gauge parameter a, and the two-loop anomalous dimension is represented in the Feynman gauge $a = 1$. So, numerically at $n_f = 3$ and $a = 1$ we find

$$\gamma^{(1)} = -4, \quad \gamma^{(2)} \approx -188.24. \tag{2.35}$$

In the leading logarithmic approximation and to the one-loop accuracy, the coefficient $\mathcal{K}_J$ is given by the expression

$$\mathcal{K}_J(\alpha_s, \mu_{\rm soft}, \mu_{\rm hard}) = \left(\frac{\alpha_s(\mu_{\rm hard})}{\alpha_s(\mu_{\rm soft})}\right)^{\frac{\gamma^{(1)}}{2\beta_0}}, \tag{2.36}$$

where $\beta_0 = 11N_c/3 - 2n_f/3 = 9$. To evaluate the two-loop expression for C_J we have to know sub-leading corrections in the first α_s order in addition to the anomalous dimensions. These corrections are not available yet, so we restrict ourselves by the one-loop accuracy.

Further, we have to determine the normalization point for the NRQCD estimates $\mu_{\rm soft}$. We put it to the average momentum transfer inside the doubly heavy diquark, so that $\mu_{\rm soft}^2 = 2m_{QQ'}T_{diq}$, where T_{diq} denotes the kinetic energy in the system of two heavy quarks, which is phenomenologically independent of the quark flavors and approximately equal to 0.2 GeV. Then, the coefficients $\mathcal{K}_J$ are equal to

$$\mathcal{K}_{\Omega(\Xi)_{cc}} \approx 1.95, \quad \mathcal{K}_{\Omega(\Xi)_{bc}} \approx 1.52, \quad \mathcal{K}_{\Omega(\Xi)_{bb}} \approx 1.30, \tag{2.37}$$

with the characteristic uncertainty about 10% because of the variation of initial and final points $\mu_{\rm hard,soft}$.

Finally, we emphasize that the values of $\mathcal{K}_J$ do not change the estimates of baryon masses calculated in the sum rules of NRQCD. However, they are essential in the evaluation of baryon couplings, which acquire these multiplicative factors.

2.2. Numerical estimates

Evaluating the two-point sum rules, we explore the scheme of moments. We point out the well-known fact that an essential part of uncertainties is caused by the variation of heavy quark masses. In the analysis we chose the following region of mass values:

$$m_b = 4.6 - 4.7 \text{ GeV}, \quad m_c = 1.35 - 1.40 \text{ GeV}, \tag{2.38}$$

which is ordinary used in the sum rule estimates for the heavy quarkonia. Next critical point is the value of QCD coupling constant determining the coulomb-like interactions inside the doubly heavy diquark. Indeed, it stands linearly in front of the perturbative functions of diquark contributions. Thus, the introduction of α_s/v-corrections is essential for both the baryon couplings and the relative contributions of perturbative terms and condensates to the baryon masses. To decrease the uncertainty we impose the same approach to the heavy quarkonia, where it is well justified, and then, we extract the characteristic values for the heavy-heavy systems from the comparison of calculations with the current data on the leptonic constants of heavy quarkonia, which are known experimentally for $c\bar{c}$ and $b\bar{b}$ or evaluated in various approaches for $\bar{b}c$. So, our calculations give the following couplings of coulomb interactions

$$\alpha_s(b\bar{b}) = 0.37, \quad \alpha_s(c\bar{b}) = 0.45, \quad \alpha_s(c\bar{c}) = 0.60. \tag{2.39}$$

Since the squared size of diquark is two times larger than that of heavy quarkonium composed of the same heavy quarks (see the dependence of average square of relative momentum on the kinetic energy of heavy quarks), the effective coulumb constants have to be rescaled according to the equation of evolution in QCD. Since we use the one-loop approximation, we explore the evolution equation

$$\alpha_s(QQ') = \frac{\alpha_s(Q\bar{Q}')}{1 - \frac{9}{4\pi}\alpha_s(Q\bar{Q}')\ln 2}.$$

So,

$$\alpha_s(bb) = 0.45, \quad \alpha_s(bc) = 0.58, \quad \alpha_s(cc) = 0.85. \tag{2.40}$$

As for the dependence of results on inputs for the quark masses, we have to remark that so called pole masses are not well defined due to infrared problems, usually mentioned as the renormalon ambiguity [45]. Thus, it is important to fix the definition of mass [46–48].

To the given order in α_s for the NRQCD sum rules, we use the leading quark loop approximation with account for the coulomb exchange between the heavy quarks. At this stage the heavy quark masses and coulomb coupling constants are strictly fixed by the data on the charmonium and bottomonium leptonic constants and masses as described by the QCD sum rules to the same accuracy. The stability or convergency of sum rule method applied to these heavy quarkonia[7] results in the following masses of quarks:

$$m_c = 1.40 \pm 0.03 \text{ GeV}, \quad m_b = 4.60 \pm 0.02 \text{ GeV},$$

which well agree with the values of heavy quark masses defined as free of infrared contributions: the potential subtracted mass $m_b^{\rm PS} = 4.60 \pm 0.11$ GeV and the kinetic mass $m_b^{\rm kin} = 4.56 \pm 0.06$ GeV both obtained in the QCD sum rules for the bottomonium within the two-loop accuracy [46, 48].

[7] We have required that the ratio of initial moments for the spectral densities calculated over the data and in QCD sum rules was stable.

The corresponding 1S-mass defined in [47] has a slightly larger value. We think that in the leading order over α_s the PS and kinetic masses above determine the threshold of quark contribution and can be taken as the appropriately defined heavy quark masses in the calculations of characteristics for the doubly heavy baryons. The mass values are dependent of normalization point, which was chosen in the range of $1-2$ GeV. Nevertheless, we slightly enlarge the region of mass variation.

The QCD sum rules for the bottomonium and charmonium fix the values of coulomb couplings, too, since the momentum stability yields the heavy quark masses, while the leptonic constants linearly determine the corresponding values of α_s shown above (see Fig. 2.1). Note that the dependence of coulomb coupling constant on the quark contents of quarkonia well agree with the renormalization group evolution with the change of size for the system composed of two heavy quarks. The uncertainty of further estimates on the supposed values of coulomb couplings is about 5%.

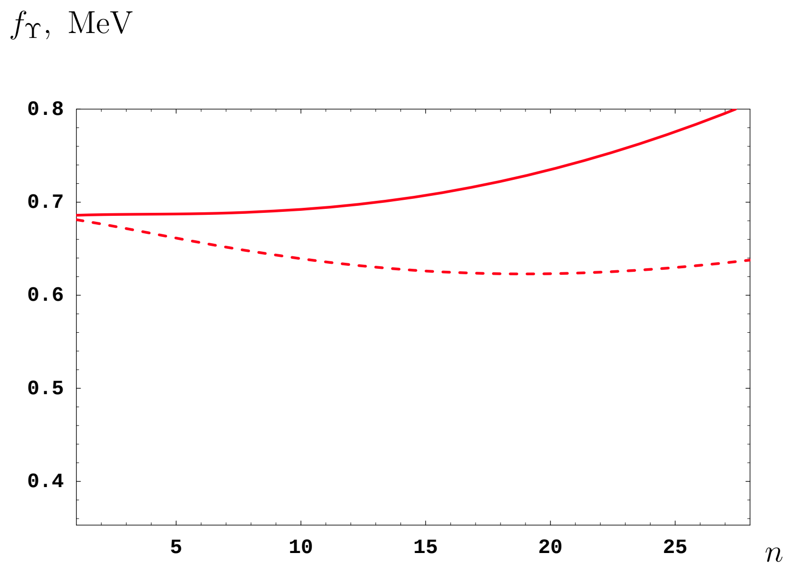

Figure 2.1. The leptonic constant of Υ in the two-point sum rules formulated in the scheme of spectral density moments. The dashed line represents the result at $m_b = 4.63$ GeV, the solid curve gives the estimate at $m_b = 4.59$ GeV.

As we have already mentioned, in the coupling constants of baryons the uncertainty connected to the coefficients for the matching the NRQCD with full QCD, is about 10%.

The dependence of estimates on the value of thershold for the continuum contribution is not so significant as on the quark masses. We fix the region of ω_{cont} as

$$\omega_{cont} = 1.3 - 1.4 \text{ GeV}. \tag{2.41}$$

For the condensates of quarks and gluons the following regions are under consideration:

$$\begin{aligned} \langle \bar{q} q \rangle &= -(250 - 270 \text{ MeV})^3, \\ m_0^2 &= 0.75 - 0.85 \text{ GeV}^2, \\ \langle \tfrac{\alpha_s}{\pi} G^2 \rangle &= (1.5 - 2) \cdot 10^{-2} \text{ GeV}^4. \end{aligned} \tag{2.42}$$

The main source of uncertainties in the ratios of the baryonic couplings is the ratio of the condensates with the strange and light quarks. We use $\langle\bar{s}s\rangle/\langle\bar{q}q\rangle = 0.8 \pm 0.2$ that corresponds to the variations of sum $(m_u + m_d)[1\,\mathrm{GeV}] = 12 \div 14$ MeV [49].

We suppose the strange quark mass equal to $m_s = 150 \pm 30$ MeV, that is the wide-accepted estimate consistent with both the sum rules and the quark current algebra, wherein the current mass of quark operates.

So, we have described the set of parameters entering the scheme of calculations.

Fig. 2.2 represents the calculated difference of masses extracted from the F_1 and F_2 correlators[8] for the baryon Ξ_{bc} (we have not shown the similar figures for the Ξ_{cc} and Ξ_{bb} baryons, since they qualitatively and quantitatively repeat the picture clearly given by Fig. 2.2).

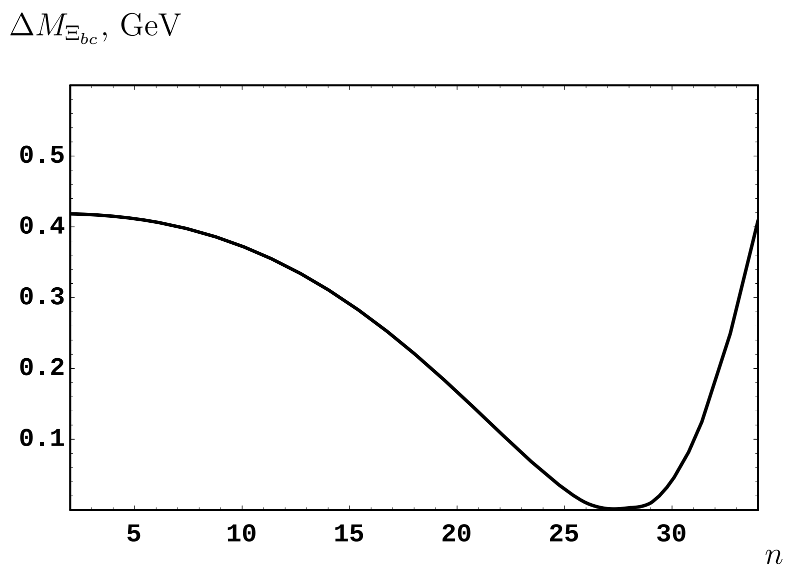

Figure 2.2. The difference between the Ξ_{bc}-baryon masses calculated in the NRQCD sum rules for the form-factors F_1 and F_2 in the scheme of moments for the spectral densities.

We certainly see that at low numbers of moments for the spectral densities, the baryon-diquark mass difference can be evaluated as

$$\bar{\Lambda} = 0.40 \pm 0.03\ \mathrm{GeV}, \tag{2.43}$$

which is quite a reasonable value, being in a good agreement with the estimates in the heavy-light mesons. In the region of mass difference stability we can fix the number of moment for the spectral density, say, $n = 27 \pm 1$ for Ξ_{bc}, and calculate the corresponding masses of baryons, which are equal to

$$M_{\Xi_{cc}} = 3.47 \pm 0.05\ \mathrm{GeV}, \quad M_{\Xi_{bc}} = 6.80 \pm 0.05\ \mathrm{GeV}, \quad M_{\Xi_{bb}} = 10.07 \pm 0.09\ \mathrm{GeV}, \tag{2.44}$$

[8] In these figures we have fixed the value of gluon condensate $\langle\frac{\alpha_s}{\pi}G^2\rangle = 1.7 \cdot 10^{-2}$ GeV^4 and arranged m_0^2 in the above region to reach zero differences between the masses, though the variation of parameters leads to errors in the estimates quoted below.

where we do not take into account the spin-dependent splitting caused by the α_s-corrections to the heavy-light interactions, which are not available yet. The uncertainties in the mass values are basically given by the variation of heavy quark masses. The convergency of NRQCD sum rules allows one to improve the accuracy of estimates in comparison with the previous analysis in full QCD [38]. The obtained values are in agreement with the calculations in the framework of nonrelativistic potential models (see Chapter 1).

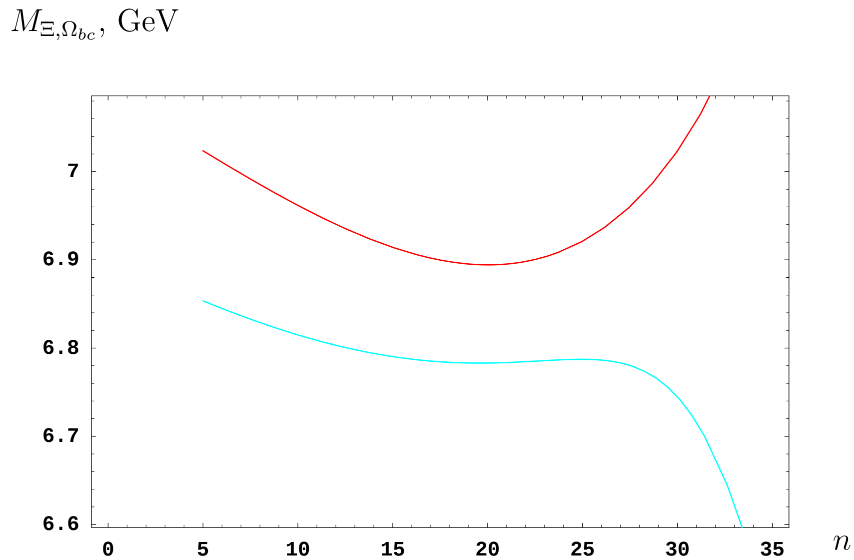

Figure 2.3. The Ξ_{bc} (lower curve) and Ω_{bc} (upper curve) masses obtained in the NRQCD sum rules by averaging the results from two correlators $F_{1,2}$.

In the two-point sum rules for the mass of Ω_{bc} (the conclusions for other doubly heavy baryons are similar) we can observe the stability of estimates with respect to changing the moment numbers in both correlators F_1 and F_2. We suppose that this fact is connected to the destroying of mentioned factorization for the correlators of diquark and baryon in the perturbative limit in contrast to the case of Ξ-baryons. The stability regions for F_1 and F_2 do not coincide because the contributions of higher dimension operators become valuable at the different numbers of moments. However, the quantity

$$\frac{1}{2}\left(M_1[n] + M_2[n]\right)$$

has the larger stability region, and we explore this fact to determine the Ω baryons masses as well as that of Ξ baryons (see Fig. 2.3). Thus, in the present review we consider two criteria for the stability of baryon mases: the first is based on the study of mass difference obtained from two correlators $F_{1,2}$, the second investigates the half sum of masses extracted from two correlators. The second way is especially reliable for the doubly heavy baryons with the strangeness, because the both correlators have the stability regions at various numbers of moments. In this way, the difference between the masses in two correlators at the stable points determines the accuracy of estimates in the framework of NRQCD sum rules.

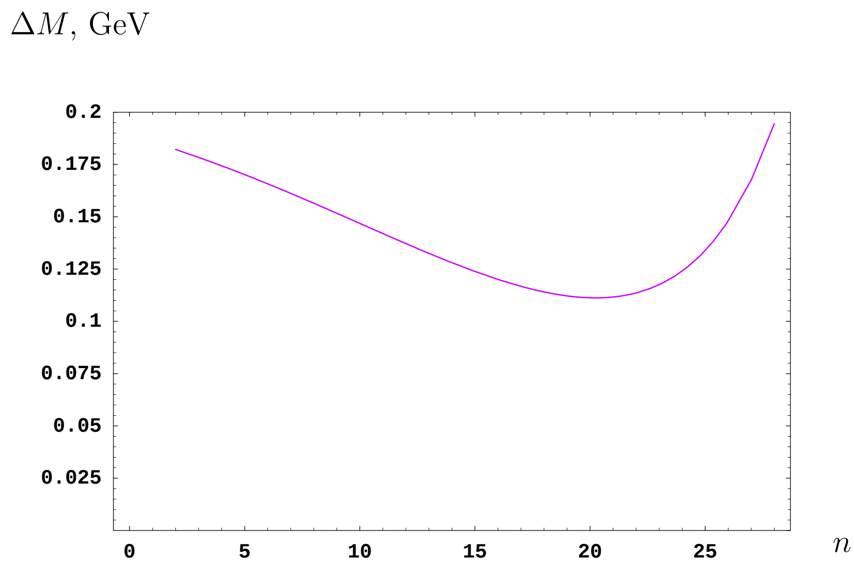

Figure 2.4. The mass difference $\Delta M = M_{\Omega_{bc}} - M_{\Xi_{bc}}$ obtained from the results shown in Fig. 2.3.

The second method results in the following values of baryon masses:

$$\begin{array}{rclrcl} M_{\Omega_{cc}} &=& 3.65 \pm 0.05 \;\; \text{GeV}, & M_{\Xi_{cc}} &=& 3.55 \pm 0.06 \;\; \text{GeV}, \\ M_{\Omega_{bc}} &=& 6.89 \pm 0.05 \;\; \text{GeV}, & M_{\Xi_{bc}} &=& 6.79 \pm 0.06 \;\; \text{GeV}, \\ M_{\Omega_{bb}} &=& 10.09 \pm 0.05 \;\; \text{GeV}, & M_{\Xi_{bb}} &=& 10.00 \pm 0.06 \;\; \text{GeV}, \end{array} \tag{2.45}$$

We can see that the estimates of $\Xi_{QQ'}$ masses in both methods of (2.45) and (2.44) well agree with each other.

Then, we investigate the difference between the masses of doubly heayy baryons with strangeness and wothout it: $1/2((M_{1,\Omega} + M_{2,\Omega}) - (M_{1,\Xi} + M_{2,\Xi}))$, shown in Fig. 2.4. In our scheme of baryon masses determination this quantity has the meaning of average difference between the masses, for which we observe a wide interval of stability indicating a good systematic accuracy of estimate. We have obtained

$$\Delta M = M_{\Omega_{bb}} - M_{\Xi_{bb}} = M_{\Omega_{cc}} - M_{\Xi_{cc}} = M_{\Omega_{bc}} - M_{\Xi_{bc}} = 100 \pm 30 \text{ MeV}.$$

Figs. 2.5, 2.6 show the dependence of baryon couplings calculated in the moment scheme of NRQCD sum rules for the doubly heavy baryons with the strangeness and without it, respectively. Numerically, we find

$$\begin{array}{rclrcl} |Z_{\Omega_{cc}}|^2 &=& (10.0 \pm 1.2) \cdot 10^{-3} \;\; \text{GeV}^6, & |Z_{\Xi_{cc}}|^2 &=& (7.2 \pm 0.8) \cdot 10^{-3} \;\; \text{GeV}^6, \\ |Z_{\Omega_{bc}}|^2 &=& (15.6 \pm 1.6) \cdot 10^{-3} \;\; \text{GeV}^6, & |Z_{\Xi_{bc}}|^2 &=& (11.6 \pm 1.0) \cdot 10^{-3} \;\; \text{GeV}^6, \\ |Z_{\Omega_{bb}}|^2 &=& (6.0 \pm 0.8) \cdot 10^{-2} \;\; \text{GeV}^6, & |Z_{\Xi_{bb}}|^2 &=& (4.2 \pm 0.6) \cdot 10^{-2} \;\; \text{GeV}^6. \end{array} \tag{2.46}$$

In Fig. 2.7 we present the sum rules results for the ratio of baryonic constants $|Z_{\Omega_{bc}}|^2/|Z_{\Xi_{bc}}|^2$. We have also found

$$|Z_{\Omega_{bc}}|^2/|Z_{\Xi_{bc}}|^2 = |Z_{\Omega_{cc}}|^2/|Z_{\Xi_{cc}}|^2 = |Z_{\Omega_{bb}}|^2/|Z_{\Xi_{bb}}|^2 = 1.3 \pm 0.2.$$

$|Z_{\Omega_{bc}}|^2$, GeV^6

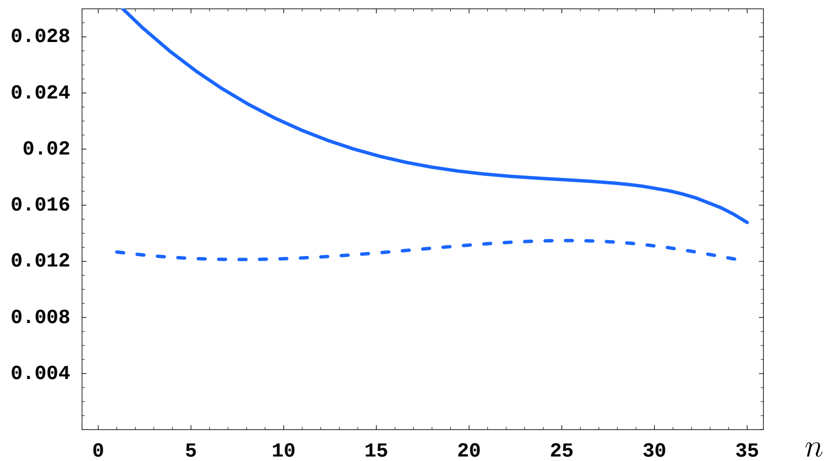

Figure 2.5. The couplings $|Z^{(1,2)}_{\Omega_{bc}}|^2$ calculated in the NRQCD sum rules for the formfactors F_1 and F_2 (solid and dashed lines, correspondingly) in the scheme of moments for the spectral densities.

The uncertainty of this result as was mentioned above is mainly connected with the pourly known ratio of $\langle \bar{s}s\rangle/\langle \bar{q}q\rangle = 0.8 \pm 0.2$.

We can see in figures that the region of stability for the baryonic constants coincides with the region of stability for the average mass described above.

For the sake of comparison, we derive the relation between the baryon coupling and the wave function of doubly heavy baryon evaluated in the framework of potential model, where we have used the approximation of quark-diquark factorization. So, we find

$$|Z^{\rm PM}| = 2\sqrt{3}|\Psi_d(0)\cdot \Psi_{l,\,s}(0)|, \tag{2.47}$$

where $\Psi_d(0)$ and $\Psi_{l,\,s}(0)$ denote the wave functions at the origin for the doubly heavy diquark and light (strange) quark-diquark systems, respectively. In the approximation used, the values of $\Psi(0)$ were calculated in the potential by Buchmüller and Tye [13], so that

$$\begin{aligned}
\sqrt{4\pi}\;|\Psi_l(0)| &= 0.53\ {\rm GeV}^{3/2}, \quad \sqrt{4\pi}\;|\Psi_s(0)| = 0.64\ {\rm GeV}^{3/2},\\
\sqrt{4\pi}|\Psi_{cc}(0)| &= 0.53\ {\rm GeV}^{3/2}, \quad \sqrt{4\pi}|\Psi_{bc}(0)| = 0.73\ {\rm GeV}^{3/2},\\
\sqrt{4\pi}|\Psi_{bb}(0)| &= 1.35\ {\rm GeV}^{3/2}.
\end{aligned}$$

In the static limit of potential models, these parameters result in the estimates

$$\begin{aligned}
|Z^{\rm PM}_{\Omega_{cc}}|^2 &= 8.8\cdot 10^{-3}\ {\rm GeV}^6, & |Z^{\rm PM}_{\Xi_{cc}}|^2 &= 6.0\cdot 10^{-3}\ {\rm GeV}^6,\\
|Z^{\rm PM}_{\Omega_{bc}}|^2 &= 1.6\cdot 10^{-2}\ {\rm GeV}^6, & |Z^{\rm PM}_{\Xi_{bc}}|^2 &= 1.1\cdot 10^{-2}\ {\rm GeV}^6,\\
|Z^{\rm PM}_{\Omega_{bb}}|^2 &= 5.6\cdot 10^{-2}\ {\rm GeV}^6, & |Z^{\rm PM}_{\Xi_{bb}}|^2 &= 3.9\cdot 10^{-2}\ {\rm GeV}^6.
\end{aligned} \tag{2.48}$$

The estimates in the potential model (2.48) are close to the values obtained in the sum rules of NRQCD (2.46). We also see that the SU(3)-flavor splitting for the baryonic constants $|Z_\Omega|^2/|Z_\Xi|^2$ is determined by the ratio $|\Psi_s(0)|^2/|\Psi_l(0)|^2 = 1.45$ which is in agreement with the sum rules result.

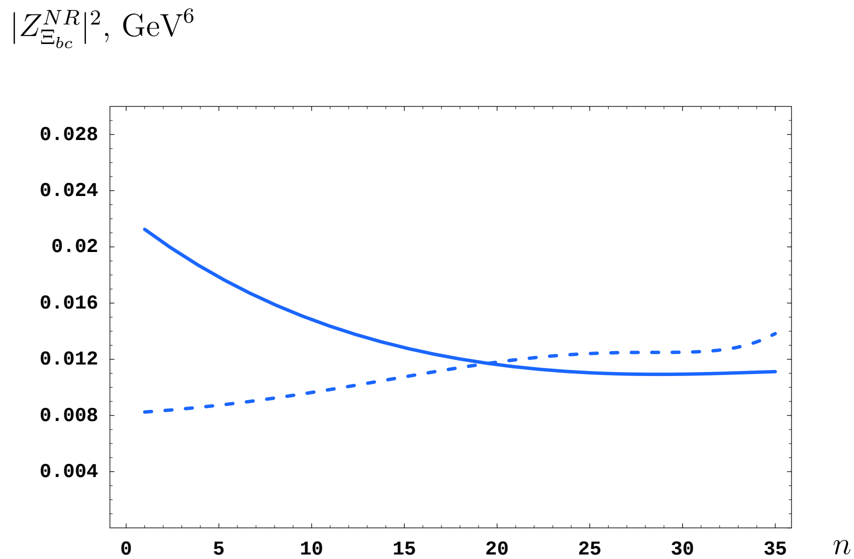

Figure 2.6. The couplings $|Z_{\Xi_{bc}}^{(1,2)}|^2$ calculated in the NRQCD sum rules for the formfactors F_1 and F_2 (solid and dashed lines, correspondingly) in the scheme of moments for the spectral densities.

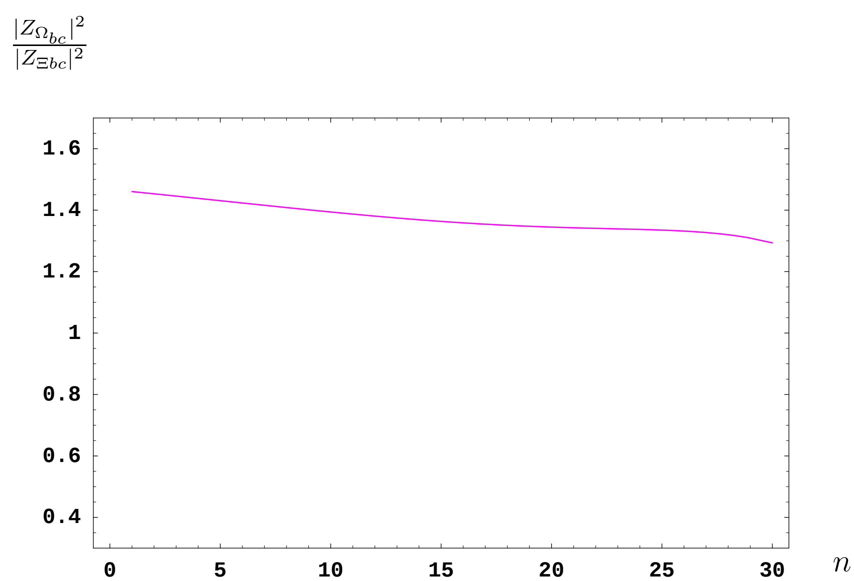

Figure 2.7. The ratio $\frac{|Z_{\Omega_{bc}}|^2}{|Z_{\Xi bc}|^2}$ calculated in the NRQCD sum rules in the scheme of moments for the spectral densities at $\langle \bar{s}s\rangle / \langle \bar{q}q\rangle = 0.8$.

The values obtained in the NRQCD sum rules have to be multiplied by the Wilson coefficients coming from the expansion of full QCD operators in terms of NRQCD fields, as they have been estimated by use of corresponding anomalous dimensions. This procedure results in the estimates

$$\begin{aligned}
|Z_{\Omega_{cc}}|^2 &= (38 \pm 5) \cdot 10^{-3}\ \mathrm{GeV}^6,\ \ |Z_{\Xi_{cc}}|^2 = (27 \pm 3) \cdot 10^{-3}\ \mathrm{GeV}^6, \\
|Z_{\Omega_{bc}}|^2 &= (36 \pm 4) \cdot 10^{-3}\ \mathrm{GeV}^6,\ \ |Z_{\Xi_{bc}}|^2 = (27 \pm 3) \cdot 10^{-3}\ \mathrm{GeV}^6, \\
|Z_{\Omega_{bb}}|^2 &= (10 \pm 1) \cdot 10^{-2}\ \mathrm{GeV}^6,\ \ |Z_{\Xi_{bb}}|^2 = (70 \pm 8) \cdot 10^{-3}\ \mathrm{GeV}^6.
\end{aligned} \tag{2.49}$$

Thus, we have got the reliable estimates for the masses and coupling constants of baryons containing two heavy quarks in the framework of NRQCD sum rules.

2.3. Discussion

We have considered the NRQCD sum rules for the two-point correlators of baryonic currents with two heavy quarks. The nonrelativistic approximation for the heavy quark fields allows us to fix the structure of baryonic currents and to take into account the coulomb-like interactions inside the doubly heavy diquark. Moreover, we have introduced into the consideration the operators of higher dimensions, which are responsible for the quark-gluon condensates in order to reach the convergency of the sum rule method for two scalar correlation functions. To the leading approximation, including the perturbative term and the contributions of quark and gluon condensates, the correlators of three-quarks state and the doubly heavy diquark are factorized in separate functions, so that the sum rules result in the different values of masses and couplings. This fact indicates the divergency of approach unless the product of quark and gluon condensates and the mixed condensate are taken into account. Then, the interaction of two heavy quarks and light quark destroy the factorization, which allows one to get meaningful estimates of masses and couplings. Moreover, we have also calculated the binding energy of doubly heavy diquark, which is in a good agreement with the estimates in the framework of potential models. In the doubly heavy baryons with the strangeness the factorization of diquark correlator is already destroyed in the approximation of quark loop in the perturbative QCD, so that the better convergency of sum rule approach takes place. So, the both correlators have the intervals of stability under the variation of number for the moment of spectral density. We have calculated the splitting of masses between the strange baryons $\Omega_{QQ'}$ and baryons with the massless light quark $\Xi_{QQ'}$. We have also estimates the ratio of baryonic coupling constants $|Z_{\Omega_{QQ'}}|^2/|Z_{\Xi_{QQ'}}|^2$ for the baryons under consideration.

Thus, the NRQCD sum rules allow us to improve the analysis of masses and couplings for the doubly heavy baryons and to obtain reliable results.

Chapter 3. Production processes

For the production of baryons with two heavy quarks $\Xi_{QQ'}$ a small ratio of Λ/m_Q and, hence, a small value of quark-gluon coupling constant in QCD $\alpha_s \sim 1/ln(m_Q/\Lambda) \ll 1$ allow us not only to consider the production of two pairs of heavy quarks $Q\bar{Q}$ and $Q'\bar{Q}'$ in the framework of perturbative QCD, but also to factorize the contributions caused by the perturbative production of heavy quarks and the nonperturbative forming of heavy diquark by the quarks enetering the $\Xi_{QQ'}$-baryons. So, in order to calculate the cross sections for the production of S-wave Ξ_{bc} states at the Z-boson peak we have to evaluate the matrix elements for the associated production of $b\bar{b}$ and $c\bar{c}$ pairs with the anti-triplet color state of bc pair having got a definite sum of quark spins ($S = 0, 1$), while the quarks are assigned to the same velocity equal to the velocity of diquark composed by these quarks. Then we have to multiply these matrix elements by a nonperturbative factor determined by some spectroscopic characteristics of bound state, i.e. by both the wave function of diquark, that gives the probability to find the quarks at short distances between them inside the bound state, and the quark masses. This approach is justified due to the following: the characteristic virtualities of heavy quarks inside the heavy diquark are much less than their masses, since the heavy quarks nonrelativistically move in the bound state, while the quark virtualities are about their masses in the hard production. Therefore, considering the production of Ξ_{bc}, we can suppose that the quarks b and c are on the mass shell in the diquark, and they are at rest with respect to each other. Thus, after the isolation of nonperturbative factor the analysis of Ξ_{bc}-baryon production is reduced to the consideration of matrix elements calculated in the perturbative QCD, if we suppose that the total and differential cross sections of baryon repeat the corresponding quantities for the doubly heavy diquark.

Note, first of all, that in the electromagnetic and strong interactions of colliding particles the associated production of two heavy quark pairs necessary for the hadronization into $\Xi_{QQ'}$, can be done provided the leading order of perturbative QCD for such processes has the additional factor of α_s^2 in comparison with the leading order for the production of single pair of heavy quarks $Q\bar{Q}$, so that $\sigma(\Xi_{QQ'})/\sigma(Q\bar{Q}) \sim \alpha_s^2|\Psi(0)|^2/m_{Q'}^3$. This suppression causes a small relative yield of $\Xi_{QQ'}$ in comparison with the inclusive production of heavy mesons M_Q.

So, we have to analyze the leading approximation of perturbative QCD for the production of $\Xi_{QQ'}$, that allows us to get some analytic expressions for the cross sections of $\Xi_{QQ'}$, particularly, for the fragmentation functions of both the heavy quark into the heavy diquark and the diquark into the baryon in the scaling limit of $M^2/s \to 0$. Thus, the production of $\Xi_{QQ'}$ in the regime of fragmentation can be reliably described in terms of analytic expressions, that opens new possibilities in the study of QCD dynamics, which is significant in the complete picture of heavy quark physics.

The mechanism for the production of baryons with two heavy quarks in the hadronic collisions involves the consideration of complete set of diagrams in the fourth order of perturbative QCD because the fragmentation regime does not dominate in the total cross section determined by other nonfragmentational contributions rapidly decreasing at large transverse momenta. We investigate a role of these higher twists over the transverse momentum of doubly heavy baryon in the associated hadronic production. We numerically determine the limits for the consistent use of factorization regime in the hard production of heavy quarks with the consequent fragmentation.

3.1. Production of doubly heavy baryons in e^+e^--annihilation

The analysis of mechanisms for the production of hadrons with two heavy quarks shows that the expected production yield of such hadrons with respect to the number of hadrons with a single heavy quark is of the order of $10^{-(3-4)}$. For example, at the Z^0-boson pole the number of events with heavy quarks is about 10^6, consequently, the number of hadrons with two heavy quarks is expected to be $\sim 100 - 1000$. Taking into account specific decay modes of hadrons with two heavy quarks one could expect the detection of several events with such hadrons, which makes their observation problematic at LEP.

In this section we consider the doubly charmed baryon production ($\Sigma_{cc}^{(*)}$) under the conditions of a B-factory with high luminosity $L = 10^{34}$ cm^{-2}s^{-1}, where the number of $\Sigma_{cc}^{(*)}$ is two orders of magnitude greater than the yield at the Z^0-boson pole.

3.1.1. Fragmentation mechanism

In [50] the production cross sections for $\Xi_{cc}^{(*)}$, $\Xi_{bc}^{(*)}$ and $\Xi_{bb}^{(*)}$ were evaluated in the region of heavy quark fragmentation at high energies. These estimations are based on the exact analytical calculations for the heavy quarkonium production in the QCD perturbation theory in the limit of small M^2/s ratio and nonrelativistic potential model. In [50] the cc-diquark momentum spectrum was considered to be equal to that of heavy vector quarkonium $(\bar{c}c)$[9]

$$D_{c\to cc}(z) = \frac{2}{9\pi}\,\frac{|R_{cc}(0)|^2}{m_c^3}\,\alpha_s^2(4m_c^2)F(z), \tag{3.1}$$

where

$$F(z) = \frac{z(1-z)^2}{(2-z)^6}\,(16 - 32z + 72z^2 - 32z^3 + 5z^4),$$

and $R_{cc}(0)$ is a radial wave function of the bound diquark at the origin.

Let us note that identical quarks cc in the color anti-triplet state can have the symmetrical spin wave function in the S-wave, i.e. they must be in the total spin $S = 1$ state. The normalization of the fragmentation function $D_{c\to cc}(z)$ is determined by the model dependent value of $R_{cc}(0)$. In [50] a rather rough approximation with the pure coulomb potential in the system of heavy quarks was used. This factor gives a huge uncertainty[10] in the estimation of $\Xi_{cc}^{(*)}$. Moreover, expression (3.1) obtained in the scaling limit $M^2/s \to 0$, is not justified for estimating the $\Xi_{cc}^{(*)}$ production at the B-factory, where the M^2/s ratio is not small.

We propose another method to estimate the production of hadrons containing two heavy quarks, on the basis of quark-hadron duality.

3.1.2. Calculations under the quark-hadron duality

A production cross section of the B_c-meson S-wave states at the Z^0-boson pole calculated in the fragmentation model is in a good agreement with the cross section estimations for the production of quark pair $(\bar{b}c)$ in the color singlet state with small invariant masses

$$m_b + m_c < M(\bar{b}c) < M_{\rm th} = M_B + M_D + \Delta M, \tag{3.2}$$

[9]There is a wrong additional factor of 2 in [50].

[10]The calculation of diquark wave function in the model with the Martin potential accounting for the factor of 1/2 due to the anti-triplet color state of quarks enforces the corresponding factor by an order of magnitude.

Table 3.1. The production cross sections of η_c and ψ mesons in e^+e^- annihilation at the B-factory.

meson	σ, pb
$\eta_c(1S)$	0.025
$\eta_c(2S)$	0.003
$\psi(1S)$	0.055
$\psi(2S)$	0.010

where $\Delta M = 0.5 - 1$ GeV.

In the same range of duality (3.2) the bc-diquark production cross section is approximately equal to that of $\bar{b}c$-pair. Selecting the color anti-triplet state (bc) and multiplying by the factor of 2/3, we obtain the estimate for the the $\Xi_{bc}^{(*)}$-baryon production cross section $\sigma(\Xi_{bc}^{(*)})/\sigma(b\bar{b}) \simeq 6 \cdot 10^{-4}$, i.e. 6 times greater than the estimate made in [50] for the $1S$-state production. This difference is caused, first, by taking into account the contribution by higher excitations of diquark in the framework of quark-hadron duality and, second, by the strong suppression due to supposed low value of $R_{bc}(0)$, evidently underestimated in the pure coulomb approximation.

Let us consider the $\Xi_{cc}^{(*)}$-baryon production at the energy of B-factory ($\sqrt{s} = 10.58$ GeV). Remember that expression (3.1) may not be used at the given energy because the power corrections over M^2/s are substantial. The method of calculations in the leading order of QCD perturbation theory was described in [51–54].

In the method of quark-hadron duality the cross section for the associated production of quarkonium bound states can be estimated by using the formula

$$\sum_{nL,J} \sigma(e^+e^- \to (nL(c\bar{c})_J)c\bar{c}) = \int_{M_i}^{M_{\rm th}} dM_{c\bar{c}} \, \frac{d\sigma(e^+e^- \to (\bar{c}c)_{singlet}c\bar{c})}{dM_{c\bar{c}}}, \tag{3.3}$$

where $M_i = 2m_c$ is the kinematical threshold of $c\bar{c}$-pair production, $M_{\rm th} = 2M_D + \Delta M$, $\Delta M = 0.5 - 1$ GeV. In Table 3.1 the results for the numerical calculations of the QCD perturbation theory diagrams are presented for the production of the bound $1S$- and $2S$-levels of charmonium at the energy $\sqrt{s} = 10.58$ GeV and $\alpha_s = 0.2$. The values of the radial wave functions at the origin $R_{nS}(0)$ have been determined from the experimental data on the lepton decay widths of charmonia $\psi(nS)$ [23]. As we can conclude from Table 3.1, below the threshold for the decay into the pair of $D\bar{D}$ mesons the sum over the S-wave states of charmonia is equal to

$$\sigma(\sum \eta_c, \psi) = 0.093 \ \ \text{pb}. \tag{3.4}$$

Note that the ratio of the vector and pseudoscalar state yields at the energy under consideration is equal to $\omega_V/\omega_P \simeq 2.2$ in contrast to the value $\omega_V/\omega_P \simeq 1$ obtained in the fragmentation mechanism [50].

Our estimations of the integral in the r.h.s. of expression (3.3) give

$$\sigma_{c\bar{c}}(\Delta M = 0.5 \text{ GeV}) = 0.093 \ \ \text{pb}, \tag{3.5}$$

$$\sigma_{c\bar{c}}(\Delta M = 1 \text{ GeV}) = 0.110 \ \ \text{pb}, \tag{3.6}$$

where we set $m_c = 1.4$ GeV.

From (3.4) and (3.5), (3.6) we see that the relation of quark-hadron duality (3.3) is well satisfied for the bound states of $(\bar{c}c)$.

Further, the calculations show that the invariant mass distributions for the $c\bar{c}$ and cc pairs coincide with each other in the region of small invariant masses. Hence, in the same duality region, the estimates for the production cross sections of $c\bar{c}$ pair and cc-diquark are approximately equal to each other, so that comparing with (3.5) and (3.6) we have

$$\begin{aligned} \sigma_{cc}(\Delta M = 0.5 \text{ GeV}) &= 0.086 \text{ pb}, &(3.7)\\ \sigma_{cc}(\Delta M = 1 \text{ GeV}) &= 0.115 \text{ pb}. &(3.8) \end{aligned}$$

Selecting the anti-triplet color state, we can obtain the summed total cross section for the production of $\Xi_{cc}^{(*)}$ baryons

$$\sigma(\Xi_{cc}^{(*)}) = (70 \pm 10) \cdot 10^{-3} \text{ pb}, \tag{3.9}$$

so that the yield fraction of doubly charmed baryons approximately equals

$$\sigma(\Xi_{cc}^{(*)})/\sigma(c\bar{c}) = 7 \cdot 10^{-5}. \tag{3.10}$$

For the luminosity equal to $L = 10^{34}\text{cm}^{-2}\text{s}^{-1}$ the number of events with the production of $\Xi_{cc}^{(*)}$ is equal to $N(\Xi_{cc}^{(*)}) = 7 \cdot 10^3$ per year, so it is by two orders of magnitude greater than the yield of $\Xi_{cc}^{(*)}$ at LEP.

In ref. [55] the distribution over the cc-diquark momentum is presented for the conditions of anti-symmetric collider KEK.

Thus, in this section we have presented the calculations of the doubly charmed $\Xi_{cc}^{(*)}$ baryon production on the basis of the quark-hadron duality in the leading order of QCD perturbation theory. We have evaluated the $\Xi_{cc}^{(*)}$ production cross section at the energy of B factory, where the fragmentation model [50] does not work.

The main theoretical uncertainty in the estimations for the production cross section of the double charmed baryons is related with the description of the process for the heavy cc-diquark hadronization. First of all, a considerable fraction of the diquarks (1/3) is produced in the color sixtet state and can transmit into both the exotic four quark states $(cc\bar{q}\bar{q})$ and the DD-meson pair. As in ref. [50], we assume that color anti-triplet state hadronizes into the $\Xi_{cc}^{(*)}$ baryon with 100% probability. Thus, at the B-factory one could expect 10^4 events per year with the production of $\Xi_{cc}^{(*)}$ at the luminosity $L = 10^{34}\ ^{-2}\ ^{-1}$.

3.1.3. Exclusive production of diquark pairs

Close to the threshold for the production of doubly heavy baryons in e^+e^- annihilation a significant contribution could be given by the pair production. In order to estimate a yield of such kind events the calculation of cross sections was done in [56] for the exclusive production of doubly heavy diquark pairs. Supposing the 100% probability for the diquark fragmentation into the baryons we can put the pair yield equal to the yield of baryons. Authors of [56] considered both the axial-vector states and the scalar ones for the S-wave diquarks, so that the amplitudes, differential and total cross sections were presented for the scalar-scalar, scalar-vector and vector-vecotr pairs. The details are given in the original ref. [56]. For the representativity we show the expression for the total cross section of scalar pairs versus the square of total energy s

$$\sigma_{ss} = 256\pi^3 \frac{f_{ss}^2}{9s^3}\, |\Psi_s(0)|^4 \left(1 - \frac{4M^2}{s}\right)^{3/2}, \tag{3.11}$$

where $\Psi_s(0)$ is the wave function of diquark at the origin, and the formfactor has the form

$$f_{ss} = \alpha_s \alpha_{em} M \left[\left(\frac{q_2}{m_1^2} + \frac{q_1}{m_2^2} \right) - \frac{2M^2}{s} \left(\frac{q_2 m_2}{m_1^3} + \frac{q_1 m_1}{m_2^3} \right) \right], \tag{3.12}$$

so that $q_{1,2}$ are the heavy quark charges, $m_{1,2}$ are their masses, and $M = m_1 + m_2$.

Numerical estimates show that the production of axial vector pairs dominates, so that in comparison with the yield of heavy quark pairs the fraction of diquark pairs is about $(2-6)\cdot 10^{-6}$. This fact implies that, say, at the B factory among the charm events one should try to observe the events with the production of doubly charmed baryon pairs yielding a 10% fraction of their inclusive production.

3.2. Perturbative fragmentation of diquark

In this section we investigate the production of baryons in the fragmentation of vector and scalar particles interacting with quarks. From the QCD point of view the doubly heavy diquark of small size is a local color-triplet field, therefore, the results of this section can be used for the calculation of fragmentation into the baryons with doubly heavy vector and scalar diquarks. In this approach we explore the perturbative QCD for the calculation of hard amplitude in the fragmentation, which is factorized in front of soft amplitude for the forming of bound state. Sure, this method is quite accurate, if the hardness is provided by a large value of mass for the quark, which composes the hadronic state of baryon by coupling together with the diquark, for example, in the fragmentation of bb into bbc. However, the obtained expressions could be used as the QCD-motivated parametrizations for the processes with the light quarks, too.

The fragmentation of scalar color-triplet local field was considered in [57]. A new problem arising in the case of vector particle is a choice of the lagrangian for the vector diquark interaction with gluons. Indeed, to the lagrangian of a free vector field $-\mathrm{tr}\,[H_{\mu\nu}\bar{H}^{\mu\nu}]$, where $H_{\mu\nu} = \partial_\mu U_\nu - \partial_\nu U_\mu$, U_μ is the vector complex field with derivatives substituted by covariant ones, we can add the gauge invariant term proportional to $S^{\alpha\beta}_{\mu\nu}\mathrm{tr}\,[G^{\mu\nu}U_\beta \bar{U}_\alpha]$, where $S^{\alpha\beta}_{\mu\nu} = 1/2(\delta^\alpha_\mu\delta^\beta_\nu - \delta^\alpha_\nu\delta^\beta_\mu)$ is the tensor of spin, $G^{\mu\nu}$ is the gluon field strength tensor. It leads to the appearance of a parameter in the gluon–diquark vertex (the so-called anomalous chromo-magnetic moment). In this section we discuss the production of a 1/2-spin bound state containing the heavy vector particle at various values of this parameter.

At high transverse momenta, the dominant production mechanism for the heavy baryon bound states is the diquark fragmentation, which can be calculated in perturbative QCD [58] after the isolation of soft binding factor extracted from the nonrelativistic potential models [8]. The corresponding fragmentation function is universal for any high energy process for the direct production of baryon.

In the leading α_s-order, the fragmentation function has a scaling form, which is the initial condition for the perturbative QCD evolution caused by the emission of hard gluons by the diquark before the hadronization. The corresponding splitting function differs from that for the heavy quark because of the spin structure of gluon coupling to the diquark, which is the vector or scalar color-triplet particle.

In this work the scaling fragmentation function is calculated in the leading order of perturbation theory. The limit of infinitely heavy diquark, $m_{diq} \to \infty$, is obtained from the full QCD consideration for the fragmentation. The distribution of bound state over the transverse momentum with respect to the axis of fragmentation is calculated in the scaling limit of perturbative QCD. The splitting

kernel of the Dokshitzer–Gribov–Lipatov–Altarelli–Parisi-evolution (DGALAP) is derived, while the one-loop equations of renormalization group for the moments of fragmentation function are obtained and solved. These equations are universal, since they do not depend on whether the diquark will bound or free at low virtualities, where the perturbative evolution stops. The integrated probabilities of diquark fragmentation into the doubly heavy baryons are evaluated.

3.2.1. Fragmentation function in the leading order

The contribution of fragmentation into direct production of heavy baryon has the form

$$d\sigma[\Xi_H(p)] = \int_0^1 dz\; d\hat{\sigma}[diq(p/z), \mu]\; D_{diq\to\Xi_H}(z,\mu),$$

where $d\sigma$ is the differential cross section for the production of baryon with the 4-momentum p, $d\hat{\sigma}$ is that of the hard production of diquark with the scaled momentum p/z, and $D(z)$ is interpreted as the fragmentation function depending on the fraction of momentum carried out by the bound state z. The value of μ determines the factorization scale. In accordance with the general DGLAP-evolution, the μ-dependent fragmentation function satisfies the equation

$$\frac{\partial D_{diq\to\Xi_H}(z,\mu)}{\partial \ln\mu} = \int_z^1 \frac{dy}{y}\; P_{diq\to diq}(z/y,\mu)\; D_{diq\to\Xi_H}(y,\mu), \tag{3.13}$$

where P is the kernel caused by the emission of hard gluons by the diquark before the production of heavy quark pair. Therefore, the initial form of fragmentation function is determined by the diagram shown in Fig.3.1, and, hence, the corresponding initial factorization scale is equal to $\mu = 2m_Q$. Furthermore, this function can be calculated as an expansion in $\alpha_s(2m_Q)$. The leading order contribution is evaluated in this section.

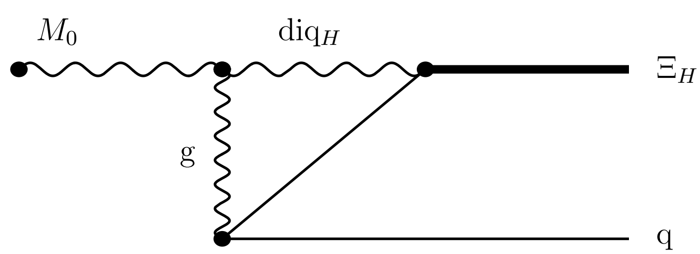

Figure 3.1. The diagram for the fragmentation of diquark diq$_H$ into the heavy baryon Ξ_H.

Consider the fragmentation diagram in the system, where the momentum of initial diquark has the form $q = (q_0, 0, 0, q_3)$ and the baryon momentum is denoted by p, so that

$$q^2 = s, \;\; p^2 = M^2.$$

In the static approximation for the bound state of diquark and heavy quark, the quark mass is expressed as $m_Q = rM$, and the diquark mass equals $m = (1-r)M = \bar{r}M$. The gluon–vector diquark vertex has the form

$$T^{VVg}_{\alpha\mu\nu} = -ig_s t^a [g_{\mu\nu}(q+\bar{r}p)_\alpha - g_{\mu\alpha}((1+æ)\bar{r}p - æq)_\nu - g_{\nu\alpha}((1+æ)q - æ\bar{r}p)_\mu], \tag{3.14}$$

where æ is the anamalous chromo-magnetic moment, t^a is the QCD group generator in the fundamental representation. The sum over the vector diquark polarizations with the momentum q ($q^2 = s$) depends on the choice of the gauge of free field lagrangian (for example, the Stueckelberg gauge), but the fragmentation function is a physical quantity and has not to depend on the gauge parameter changing the contribution of longitudinal components of the vector field. So, in a general case, the sum over polarization can be taken in the form

$$P(q)_{\mu\nu} = -g_{\mu\nu} + \frac{q_\mu q_\nu}{s}.$$

The matrix element of the fragmentation into the baryon with the spin of 1/2 has the form

$$\mathcal{M} = -\frac{2\sqrt{2\pi}\alpha_s}{9\sqrt{M^3}} \frac{R(0)}{r\bar{r}(s-m^2)^2} P(q)_{\nu\delta} P(\bar{r}p)_{\mu\eta} T^{VVg}_{\alpha\mu\nu} \rho_{\alpha\beta}\, \bar{q}\gamma^\beta(\hat{p}-M)\gamma^\eta\gamma^5\xi_H\, \mathcal{M}^\delta_0, \tag{3.15}$$

where the sum over the gluon polarization is written down in the axial gauge with $n = (1,0,0,-1)$

$$\rho_{\mu\nu}(k) = -g_{\mu\nu} + \frac{k_\mu n_\nu + k_\nu n_\mu}{k\cdot n},$$

and $k = q-(1-r)p$. The spinors ξ_H and $\bar{q}$ correspond to the baryon and heavy quark associated to the fragmentation. $\mathcal{M}_0$ denotes the matrix element for the hard production of diquark at high energy, $R(0)$ is the radial wave function at the origin. The matrix element squared and summed over the helicities of particles in the final state has the following structure:

$$|\overline{\mathcal{M}}|^2 = W_{\mu\nu} M^\mu_0 M^\nu_0.$$

In the limit of high energies $q\cdot n \to \infty$ $W_{\mu\nu}$ behaves like

$$W_{\mu\nu} = -g_{\mu\nu} W + R_{\mu\nu}, \tag{3.16}$$

where $R_{\mu\nu}$ can depend on the gauge parameters. After the separation of Lorentz structures it leads to scalar formfactor terms, which are small in comparison with W in the limit of $q\cdot n \to \infty$. Define

$$z = \frac{p\cdot n}{q\cdot n}.$$

The fragmentation function is determined by the expression [59]

$$D(z) = \frac{1}{16\pi^2}\int ds\, \theta\Big(s - \frac{M^2}{z} - \frac{m_Q^2}{1-z}\Big)\, W,$$

where W is defined in (3.16). The integral in the expression for the fragmentation function diverges logarithmically at a constant value of anamalous chromo-magnetic moment if æ does not equal -1. We consider two sets for the behaviour of anamalous chromo-magnetic moment. The first is

æ $= -1$. Here we observe that the obtained fragmentation function coincides with that for the scalar diquark up to the factor of 1/3

$$D(z) = \frac{8\alpha_s^2}{243\pi} \frac{|R(0)|^2}{M^3 r^2 \bar{r}^2} \frac{z^2(1-z)^2}{[1-\bar{r}z]^6} \quad \cdot \quad \Big\{3 + 3r^2 - (6 - 10r + 2r^2 + 2r^3)z + \\ +(3 - 10r + 14r^2 - 10r^3 + 3r^4)z^2)\Big\}, \qquad (3.17)$$

which tends to

$$\tilde{D}(y) = \frac{8\alpha_s^2}{243\pi\ y^6} \frac{|R(0)|^2}{m_Q^3} \frac{(y-1)^2}{r}\Big\{8 + 4y + 3y^2\Big\}, \qquad (3.18)$$

at $r \to 0$ and $y = (1 - (1-r)z)/(rz)$.

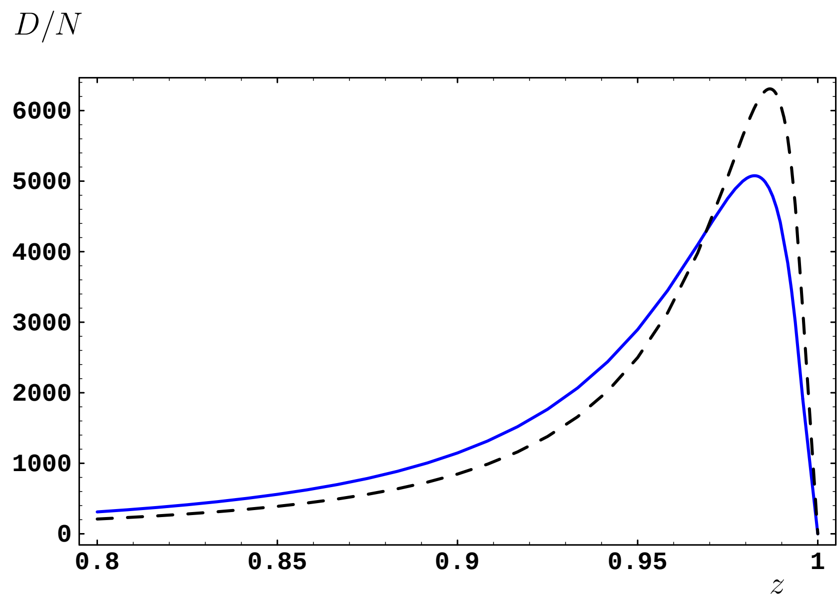

Figure 3.2. The fragmentation function of diquark into the heavy baryon, the N-factor is determined by $N = \frac{8\alpha_s^2}{243\pi} \frac{|R(0)|^2}{M^3 r^2 (1-r)^2}$, the fragmentation function at æ $= -1$ is shown by the dashed line, the fragmentation function at $1 +$ æ $= 3M^2/(s - m_{diq}^2)$ is given by the solid line ($r = 0.02$).

The limit of $\tilde{D}(y)$ is in agreement with the general consideration of $1/m$-expansion for the fragmentation function [60], where

$$\tilde{D}(y) = \frac{1}{r} a(y) + b(y).$$

The dependence of $a(y)$ on y has the same form as for the fragmentation of heavy quark into the quarkonium [59].

The consideration of fragmentation at æ $= -1 + AM^2/(s - m_{diq}^2)$ was done in [61].

The perturbative functions in the leading α_s-order are shown in Fig.3.2 at $r = 0.02$. We see hard distributions, which become softer with the evolution (see ref. [57]).

3.2.2. Transverse momentum of baryon

In the system with an infinite momentum of the fragmentating diquark its invariant mass is expressed by the fraction of longitudinal diquark momentum z and transverse momentum with respect to the fragmentation axis p_T (see Fig. 3.1) as

$$s = m^2 + \frac{M^2}{z(1-z)}[(1-(1-r)z)^2 + t^2],$$

where $t = p_T/M$. The calculation of diagram in Fig. 3.1 gives the double distribution for the fragmentation probability

$$\frac{d^2P}{ds\, dz} = \mathcal{D}(z,s),$$

where at (æ $= -1$) the function $\mathcal{D}$ has the form

$$\begin{aligned} \mathcal{D}(z,s) &= \frac{256\alpha_s^2}{81\pi}\frac{|R(0)|^2}{r^2\bar{r}^2}\frac{M^3}{[1-\bar{r}z]^2(s-m^2)^4} \\ & \Big\{r\bar{r}^2 + \bar{r}(1+r-z(1+4r-r^2))\frac{s-m^2}{M^2} - z(1-z)\Big(\frac{s-m^2}{M^2}\Big)^2\Big\}. \end{aligned} \tag{3.19}$$

The distribution of baryon over the transverse momentum can be obtained by the integration of

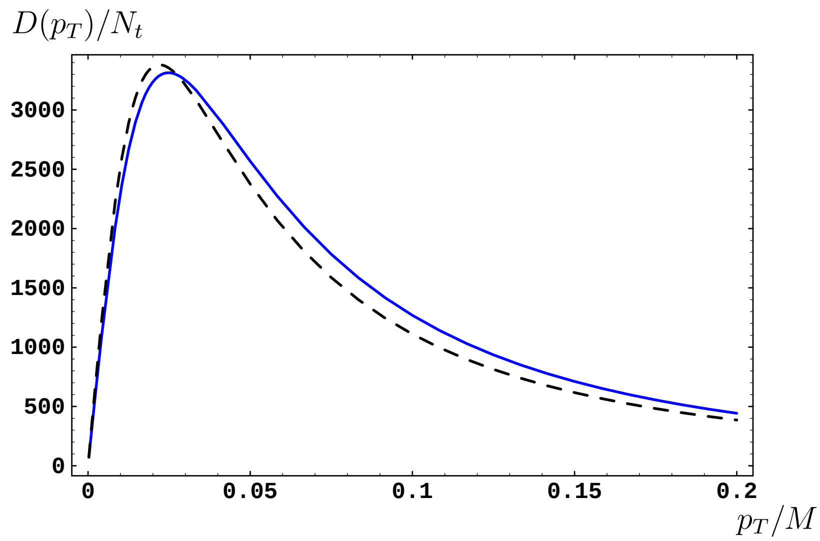

Figure 3.3. The distributions over the transverse momentum with respect to the axis of diquark fragmentation into the baryon, N_t-factor is determined by $N_t = \frac{8\alpha_s^2}{81\pi}\frac{|R(0)|^2}{M^4r^2(1-r)^7}$ at $r = 0.02$. The dashed line represents the result at æ $= -1$, the solid line does it at æ $= -1 + 3M^2/(s - m_{diq})$.

(3.19) over z

$$D(t) = \int_0^1 dz\, \mathcal{D}(z,s)\, \frac{2M^2t}{z(1-z)}.$$

Finally, we get quite a cumbersome expression presented in Appendix II. The characteristic form of distribution over the transverse momentum of baryon with respect to the axis of diquark fragmentation is shown in Fig. 3.3.

3.2.3. Hard gluon emission

The one-loop contribution can be calculated in the way described in the previous sections. This term does not depend on the part of diquark-gluon vertex with the anomalous chromo-magnetic moment, therefore the splitting kernel coincides with that for the scalar diquark. It equals

$$P_{diq\to diq}(x,\mu) = \frac{4\alpha_s(\mu)}{3\pi}\left[\frac{2x}{1-x}\right]_+, \tag{3.20}$$

where the "plus" denotes the ordinary action: $\int_0^1 dx f_+(x)\cdot g(x) = \int_0^1 dx f(x)\cdot[g(x)-g(1)]$. The diquark splitting function can be compared with that of the heavy quark

$$P_{Q\to Q}(x,\mu) = \frac{4\alpha_s(\mu)}{3\pi}\left[\frac{1+x^2}{1-x}\right]_+,$$

which has the same normalization factor at $x\to 1$.

Furthermore, multiplying the evolution equation by z^n and integrating over z, one can get from (3.13) the μ-dependence of moments $a_{(n)}$ up to the one-loop accuracy of renormalization group

$$\frac{\partial a_{(n)}}{\partial \ln\mu} = -\frac{8\alpha_s(\mu)}{3\pi}\left[\frac{1}{2}+\ldots+\frac{1}{n+1}\right] a_{(n)}, \quad n\geq 1. \tag{3.21}$$

At $n=0$ the right hand side of (3.21) equals zero, which means that the integrated probability of diquark fragmentation into the heavy baryon does not change during the evolution, and it is determined by the initial fragmentation function calculated perturbatively [57].

The solution of equation (3.21) has the form

$$a_{(n)}(\mu) = a_{(n)}(\mu_0)\left[\frac{\alpha_s(\mu)}{\alpha_s(\mu_0)}\right]^{\frac{16}{3\beta_0}\left[\frac{1}{2}+\ldots+\frac{1}{n+1}\right]}, \tag{3.22}$$

where one has used the one-loop expression for the QCD coupling constant

$$\alpha_s(\mu) = \frac{2\pi}{\beta_0\ln(\mu/\Lambda_{QCD})},$$

where $\beta_0 = 11-2n_f/3$, with n_f being the number of quark flavors with $m_q<\mu<m_{diq}$.

Relation (3.22) is universal one, since it is independent of whether the diquark is free or bound at the virtualities less than μ_0. In this paper we take into account the evolution for the fragmentation into the heavy baryon. The diquark can lose about 20% of its momentum before the hadronization [57].

3.2.4. Integrated probabilities of fragmentation

As has been mentioned above, the evolution conserves the integrated probability of fragmentation which can be calculated explicitly

$$\int dz\; D(z) \;=\; \frac{8\alpha_s^2}{81\pi}\,\frac{|R(0)|^2}{16m_Q^3}\,\mathrm{w}(r), \tag{3.23}$$

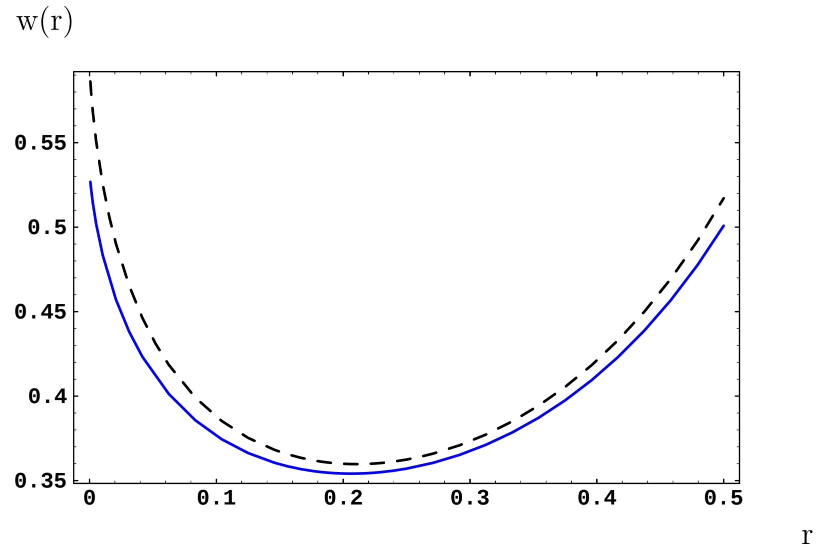

Figure 3.4. The w functions for the diquark fragmentation into the heavy baryon versus the fraction $r = m_Q/M$. The dashed curve corresponds to the case of $æ = -1$, while the solid line does to $æ = -1 + 3M^2/(s - m_{diq}^2)$.

so that

$$\mathrm{w}(r) = \frac{16[(8 + 15r - 60r^2 + 100r^3 - 60r^4 - 3r^5) + 30r(1 - r + r^2 + r^3)\ln r]}{15(1-r)^7}. \tag{3.24}$$

The $\mathrm{w}(r)$ functions are shown in Fig. 3.4 at low r.

Thus, we have considered the dominant mechanism for the production of bound spin 1/2 states composed by a local color-triplet vector field with a heavy anti-quark for high energy processes at large transverse momenta, where the fragmentation contributes as the leading term. We have investigated two cases for the behaviour of anamalous chromo-magnetic moment. At $æ = -1$ we observe[11] that the obtained fragmentation function coincides with that for the scalar diquark up to a factor. In the infinitely heavy diquark limit, $D(z)$ has the form, which agrees with what expected from general consideration of $1/m$-expansion for the fragmentation function. The distribution of bound state over the transverse momentum with respect to the axis of diquark fragmentation is calculated in the scaling limit of perturbative QCD. The hard gluon corrections caused by the splitting of vector diquark has been taken into account in the perturbation theory, that has led to the corresponding one-loop equations of renormalization group for the moments of fragmentation function (see (3.21), (3.22)).

The numerical estimates show that the probabilities of fragmentation into the bound states containing the heavy vector diquark with the mass from 3 to 10 GeV depend on the effective mass of quark entering the baryon together with the diquark. In this way the ratio of yields for the baryons with the strangeness and without it, is approximately equal to $\sigma(\Omega_{QQ'})/\sigma(\Xi_{QQ'}) \approx 0.2$. Certainly, the introduction of light and strange quarks into the consideration makes the obtained results to be not strictly justified, since the calculations suggest that the constituent masses lead to the effective

[11]The expression for the fragmentation function diverges logarithmically at a constant value of anamalous chromo-magnetic moment if $æ$ is not equal to -1.

and correct description of dominant contributions caused by the infrared dynamics. However, we can use the perturbative expressions as models for the fragmentation into the hadrons containing light and strange quarks because in such processes we describe the "fast" valence degrees of freedom in the baryon. In the limit of small dispersion these degrees of freedom can be approximated by the introduction of ratio for the assigned fractions of longitudinal moment for the parton inside the hadron, while we do not provide the consideration of contribution by a soft sea of light quarks and gluons.

Another approach to the fragmentation mechanism for the production of doubly heavy baryons was considered in ref. [62], wherein the perturbative formfactors of doubly heavy and heavy-light diquarks were calculated and the heavy quark fragmentation function into the baryons was derived in the process with the production of vector diquark pairs: $Q \to H_{Q(Q'q)_{diq}} + (\bar{Q}'\bar{q})_{diq}$. In this way the estimates strictly mean the minimal expectations, since they are based on the elastic formfactor of diquark. We also believe that the hierarchy of scales $m_Q \gg m_Q \cdot v \gg \Lambda_{QCD}$ in the relevant strong interactions provides that after the hard production of heavy quark (the virtualities about m_Q) quite a fast forming of the doubly heavy diquark occures (the virtualities of the order of $m_Q \cdot v$), while a more slow hadronization of diquark into the baryons (the virtualities sloe to Λ_{QCD}) takes place in the final stage. V.Saleev analyzed the fragmentation into the thriply charmed baryon Ω_{ccc} [63] due to the cascade process of quark into the diquark and diquark into the baryon in comparison with the direct fragmentation of quark into the baryon due to the elastic production of vector diquark. Unfortumately, the general conclusion of [63] on the significant dominance of direct fragmentation is not correct, because, first, the use of elastic formfactor for the vector diquark in the cascade process leads to the double counting of suppressing factor due to the charmed quark fusion into the diquark in the fragmetation of quark into the diquark as well as in the elastic formfactor, where one again took the projection of initial quark state on the bound diquark. This procedure gives the overcoming factor of suppression $\alpha_s^2|\Psi_{cc}(0)|^2/m_c^3 \sim 10^{-3}$. Second, the idea on the forming the cc-diquark with the consequent hard production of charmed quark off diquark is not correct because the time period (the diquark size) for forming the diquark is much greater than the time interval (the compton length) for the production of charmed quark.

3.3. Hadronic production

Recent years are marked by a rapid increase of charmed particles observed in modern experiments. So, the study of about 10^6 charmed particles is expected at fixed target FNAL facilities of E831 and E781. An increase of this value by two orders of magnitude is proposed in experiments of next generation. Along with standard problems of CP-violation in the charmed quark sector and a measuring of rare decays etc., an investigation of processes with more than one $c\bar{c}$-pair production becomes actual. The production of additional $c\bar{c}$-pair strongly decreases a value of cross section for such processes. This fact must be especially taken into account in fixed target experiments, where the quark-partonic luminosities are strongly suppressed in the region of heavy mass production.

An interesting process of the mentioned kind is the doubly charmed baryon production. The doubly charmed $\Xi_{cc}^{(*)}$-baryon represents an absolutely new type of objects in comparison with the ordinary baryons containing light quarks only. The ground state of such baryon is analogous to a $\bar{Q}q$-meson, which contains a single heavy anti-quark $\bar{Q}$ and a light quark q. In the doubly heavy baryon the role of heavy anti-quark is played by the cc-diquark, which is in anti-triplet color state [64]. It has a small size in comparison with the scale of the light quark confinement. The study of mechanism for the production of these states are of interest. The ccq-baryon production

was discussed in [50, 54, 55, 65, 66]. The main problem of calculations is reduced to an evaluation of the production cross section for the diquark in the $\bar{3}$-triplet color state. One assumes further that the cc-diquark nonperturbatively transforms into the ccq-baryon with a probability close to unit.

The hadronic production of diquark is subdivided into two parts. The first stage is the hard production of two $c\bar{c}$-pairs in the processes of $gg \to c\bar{c}c\bar{c}$ and $q\bar{q} \to c\bar{c}c\bar{c}$ described by the Feynman diagrams in the fourth order of α_s coupling constant in the perurbative QCD (see Fig. 3.5).

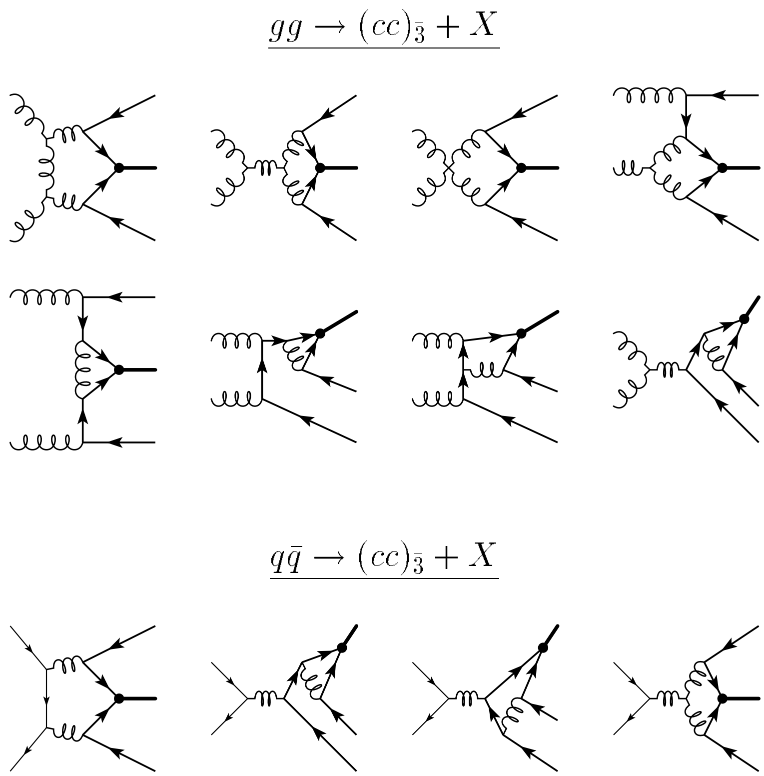

Figure 3.5. The examples of diagrams for the gluon-gluon and quark-antiquark production of cc-diquark. The initial quarks are denoted by the thin fermion lines, the final quarks are denoted by the bold fermion lines and the gluons are denoted by the helical lines.

The second step is the nonperturbative fusion of two c-quarks with a small relative momentum into the cc-diquark. For the S-wave states this process is characterized by the radial wave function at the origin, $R(0)$.

The main difference between the existing evaluations of the doubly charmed baryon cross section consists in the methods used for the hard subprocess calculation. In [67] a part of diagrams connected with the c-fragmentation into the (cc)-diquark is only used instead of the complete set of diagrams. As was shown in [65] this estimation is not absolutely correct, because it becomes true only at large transverse momenta greater than $p_T^{min} > 35$ GeV, where the fragmentation mechanism is dominant. In other kinematical regions the application of fragmentational approximation is not justified and it leads to wrong results, especially at $\sqrt{\hat{s}}$ being not much greater than p_T^{min}.

However, even after taking into account the complete set of diagrams considered in [65, 66], essential uncertainties in the estimations of the ccq-baryon production remain. The basic parameters determining these uncertainties are the values of α_s, m_c and $R_{cc}(0)$. In addition, it is not clear, to what extent the hypothesis on the hadronization of cc-diquark into the ccq-baryon with the unit probability is correct or not. The matter is that the interaction between the diquark and gluons is

not suppressed in contrast to the $c\bar{c}$-pair production in the color singlet state, when the quarkonium dissociation supposes exchange with the quark-gluonic sea by two hard gluons with virtualities, which are greater than the inverse size of quarkonium.

A decrease of the uncertainty in the ccq-baryon cross section would be possible by means of comparing the process of baryon production with the analogous process of $J/\Psi + D\bar{D}$ production[12]. The associated production of J/Ψ is described by practically the same diagrams of fourth order with the well-known wave function of J/Ψ at the origin[13]. In this way of connection to the $J/\Psi + D\bar{D}$ process we could remove the part of uncertainties following from α_s and m_c in the cc-diquark production process.

In the following sections of the paper the joint cross section calculations of these processes in $\pi^- p$ and pp interactions are performed, the description of production models for the ccq-baryons and $J/\Psi + D\bar{D}$ is given. We presents the calculation results for the production cross section of ccq-baryons and $J/\Psi + D\bar{D}$ in the fixed target experiments E781 and HERA-B as well as at the collider energies of Tevatron and LHC. We discuss possibilities of searching for the baryons $\Xi_{cc}^{(*)}$.

3.3.1. Production mechanism

As was mentioned, we suppose that the diquark production can be subdivided by two stages. On the first stage the production amplitude of four free quarks is calculated for the following processes

$$\begin{aligned} gg &\to cc\bar{c}\bar{c}, \\ q\bar{q} &\to cc\bar{c}\bar{c}. \end{aligned}$$

The calculation technique applied in this work is analogous to that for the hadronic production of B_c [53, 68], but in this case the bound state is composed by two quarks (Q_1 and Q_2) instead of the quark and anti-quark [54, 55, 65].

One assumes that the binding energy in the diquark is much less than the masses of constituent quarks and, therefore, these quarks are on the mass shells. So, the quark four-momenta are related to the (Q_1Q_2) diquark momentum $P_{(Q_1Q_2)}$ in the following way:

$$p_{Q_1} = \frac{m_{Q_1}}{M_{(Q_1Q_2)}} P_{(Q_1Q_2)}\ , \qquad p_{Q_2} = \frac{m_{Q_2}}{M_{(Q_1Q_2)}} P_{(Q_1Q_2)}, \tag{3.25}$$

where $M_{(Q_1Q_2)} = m_{Q_1} + m_{Q_2}$ is the diquark mass, m_{Q_1}, m_{Q_2} are the quark masses.

In the given approach the diquark production is described by 36 Feynman diagrams of the leading order, corresponding to the production of four free quarks with the combining of two quarks into the color anti-triplet diquark with the given quantum numbers over the Lorentz group. The latter procedure is performed by means of the projection operators

$$\mathcal{N}(0,0) = \sqrt{\frac{2M_{(Q_1Q_2)}}{2m_{Q_1}2m_{Q_2}}} \frac{1}{\sqrt{2}} \{\bar{u}_1(p_{Q_1},+)\bar{u}_2(p_{Q_2},-) - \bar{u}_1(p_{Q_1},-)\bar{u}_2(p_{Q_2},+)\} \tag{3.26}$$

[12]We calculate the $J/\psi + c\bar{c}$ production and assume that the $c\bar{c}$-pair transforms to $D\bar{D}+$ some light hadrons with the probability very close to unit. So, we neglect the production of charmed baryons as well as bound states of charmonium in the hadronization of associated $c\bar{c}$ pair.

[13]The value of $|R_\Psi(0)|$ is determined by the width of leptonic decay, $J/\Psi \to l^+l^-$ with taking into account the hard gluonic correction, so, numerically, $|R_\psi(0)| = \sqrt{\pi M/3}\tilde{f}_\psi$, where $\tilde{f}_\psi = 540$ MeV.

for the scalar state of diquark (the corresponding baryon is denoted by $\Xi'_{Q_1Q_2}(J=1/2)$);

$$\begin{aligned}
\mathcal{N}(1,-1) &= \sqrt{\frac{2M_{(Q_1Q_2)}}{2m_{Q_1}2m_{Q_2}}}\bar{u}_1(p_{Q_1},-)\bar{u}_2(p_{Q_2},-),\\
\mathcal{N}(1,0) &= \sqrt{\frac{2M_{(Q_1Q_2)}}{2m_{Q_1}2m_{Q_2}}}\frac{1}{\sqrt{2}}\{\bar{u}_1(p_{Q_1},+)\bar{u}_2(p_{Q_2},-)+\bar{u}_1(p_{Q_1},-)\bar{u}_2(p_{Q_2},+)\},\\
\mathcal{N}(1,+1) &= \sqrt{\frac{2M_{(Q_1Q_2)}}{2m_{Q_1}2m_{Q_2}}}\bar{u}_1(p_{Q_1},+)\bar{u}_2(p_{Q_2},+)
\end{aligned} \tag{3.27}$$

for the vector state of diquark (the baryons are denoted as $\Xi_{Q_1Q_2}(J=1/2)$ and $\Xi^*_{Q_1Q_2}(J=3/2)$).

To produce the quarks, composing the diquark in the $\bar{3}_c$ state, one has to introduce the color wave function as $\varepsilon_{ijk}/\sqrt{2}$, into the diquark production vertex, so that $i=1,2,3$ is the color index of the first quark, j is that of the second one, and k is the color index of diquark. For the identical quarks $Q_1=Q_2$ possessing the momenta equal to each other, the anti-triplet color state can have the summed spin $S=1$, only.

The diquark production amplitude $A_k^{Ss_z}$ is expressed through the amplitude $T_k^{Ss_z}(p_i)$ for the free quark production in kinematics (3.25) under the substitution for the product $\bar{u}_1\bar{u}_2$ by the projection operators as well as under the condition that two heavy quarks in the $\bar{3}$-color state, so that

$$A_k^{Ss_z}=\frac{R_{Q_1Q_2}(0)}{\sqrt{4\pi}}T_k^{Ss_z}(p_i), \tag{3.28}$$

where $R_{Q_1Q_2}(0)$ is the diquark radial wave function at the origin, k is the color state of diquark, S and s_z are the diquark spin and its projection on the z-axis, correspondingly.

In numerical calculations under discussion we suppose the following values of parameters:

$$\begin{aligned}
&\alpha_s=0.2,\quad m_c=1.7\ \text{GeV},\quad m_b=4.9\ \text{GeV},\\
&R_{cc(1S)}(0)=0.601\ \text{GeV}^{3/2},\quad R_{bc(1S)}(0)=0.714\ \text{GeV}^{3/2},
\end{aligned} \tag{3.29}$$

where the value of $R_{cc}(0)$ has been calculated by means of numerical solution of the Schrödinger equation with the Martin potential multiplied by the 1/2 factor caused by the color anti-triplet state of quarks instead of the singlet one.

To calculate the production cross section of diquarks composed of two c-quarks, one has to account for their identity. One can easily find, that the anti-symmetrization over the identical fermions leads to the scalar diquark amplitude equal to zero, and it results in the amplitude of the vector cc-diquark production being obtained by the substituting of equal masses in the production amplitude of vector diquark composed of two quarks with the different flavors, and taking into account the 1/2 factor for the identical quarks and anti-quarks.

In this approach we suppose that the produced diquark forms the baryon with the unit probability by catching up the light quark from the quark-antiquark sea at small p_T or having the fragmentation into the baryon at large p_T.

The typical diagrams of fourth order describing the parton processes are shown in Fig. 3.5. One can subdivide them into two groups. The first group contains the diagrams of fragmentation type, wherein the $c\bar{c}$-pair emits another one. The second group corresponds to the independent dissociation of gluons into the $c\bar{c}$-pairs with the consequent fusion into the diquark. The diagrams of second group belong to the recombination type.

As was mentioned above, the authors of some papers [67] restricted themselves by the consideration of fragmentation diagrams, only. In this way they reduced the cross-section formulae to the $c\bar{c}$-pair production cross section multiplied by the fragmentation function of c-quark into the cc-diquark.

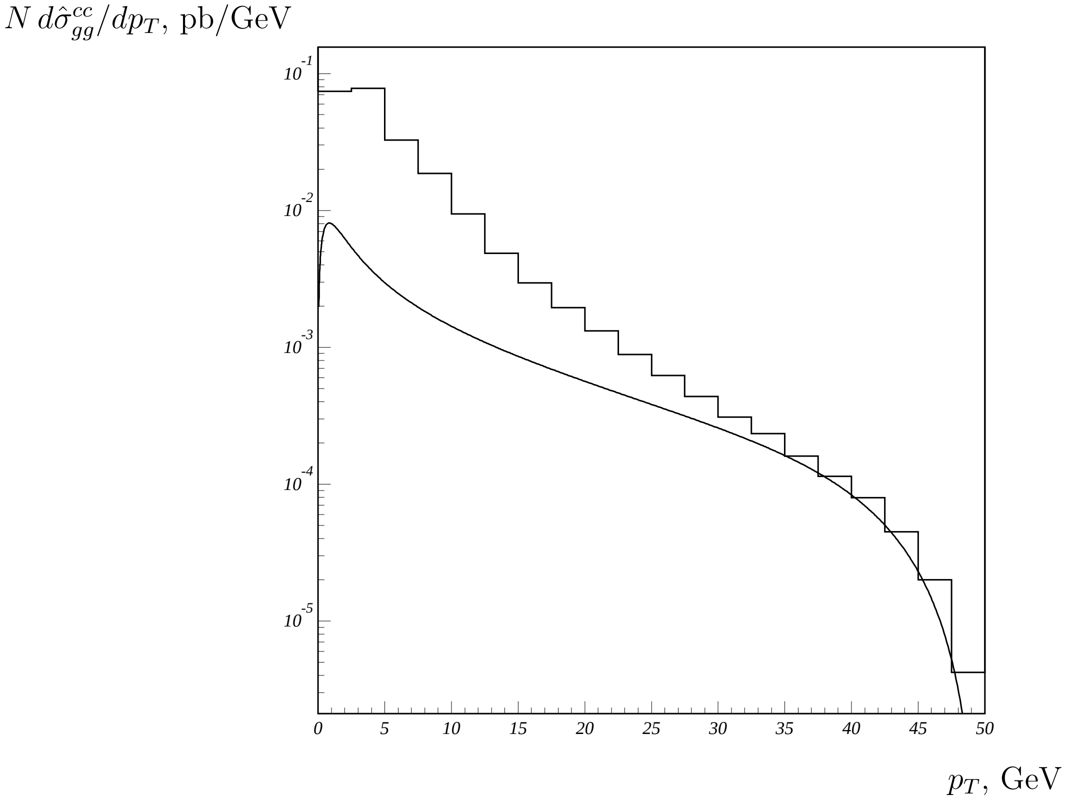

Figure 3.6. The differential cross section for the associated production of cc diquarkin the gluon-gluon subprocess at 100 GeV (histogram) in comparison with the prediction of fragmentation model (solid curve), correspondingly.

As we shown in [65], this approximation is correct only under the two following conditions: $M^2_{QQ} \ll \hat{s}$ and $p_T \gg M_{QQ}$. In other kinematical regions, the contribution of recombination diagrams dominates. The typical value of p_T, wherefrom the fragmentation begins to dominate in the production of -diquark, is $p_T > 35$ GeV (see Fig. 3.6). It is clear, that at realistic p_T one has to take into account all contributions including the recombination. For the first time, the complete set of diagrams was taken into account in [65] and confirmed in [66]. In the both papers the calculations are performed only for the gluon-gluon production, which is a rather good approximation at collider energies. For the fixed target experiments the value of total energy strongly decreases, and, hence, the values of energy in the parton subprocesses decrease too. The contribution of quark-antiquark annihilation becomes essential at fixed target energies, especially for the processes with initial valence anti-quarks. In the following consideration we take into account the quark-antiquark annihilation into four free charmed quarks in the estimation of yield for the doubly charmed baryon.

3.3.2. Doubly charmed baryon production in fixed target experiments

In Figs 3.7 and 3.8 the calculation results for the total cross section of the diquark-production subprocesses versus the total energy $\sqrt{\hat{s}}$ for the given values of α_s, m_c and $R_{cc}(0)$. We have also

shown the parametrizations for the dependencies of total cross sections versus the process energy for the cc-diquark as given by

$$\hat{\sigma}_{gg}^{(cc)} = 213. \left(1 - \frac{4m_c}{\sqrt{\hat{s}}}\right)^{1.9} \left(\frac{4m_c}{\sqrt{\hat{s}}}\right)^{1.35} \quad \text{pb}, \tag{3.30}$$

$$\hat{\sigma}_{q\bar{q}}^{(cc)} = 206. \left(1 - \frac{4m_c}{\sqrt{\hat{s}}}\right)^{1.8} \left(\frac{4m_c}{\sqrt{\hat{s}}}\right)^{2.9} \quad \text{pb}. \tag{3.31}$$

We have to stress that the numerical coefficients depend on the model parameters, so that $\hat{\sigma} \sim \alpha_s^4 |R(0)|^2/m_c^5$.

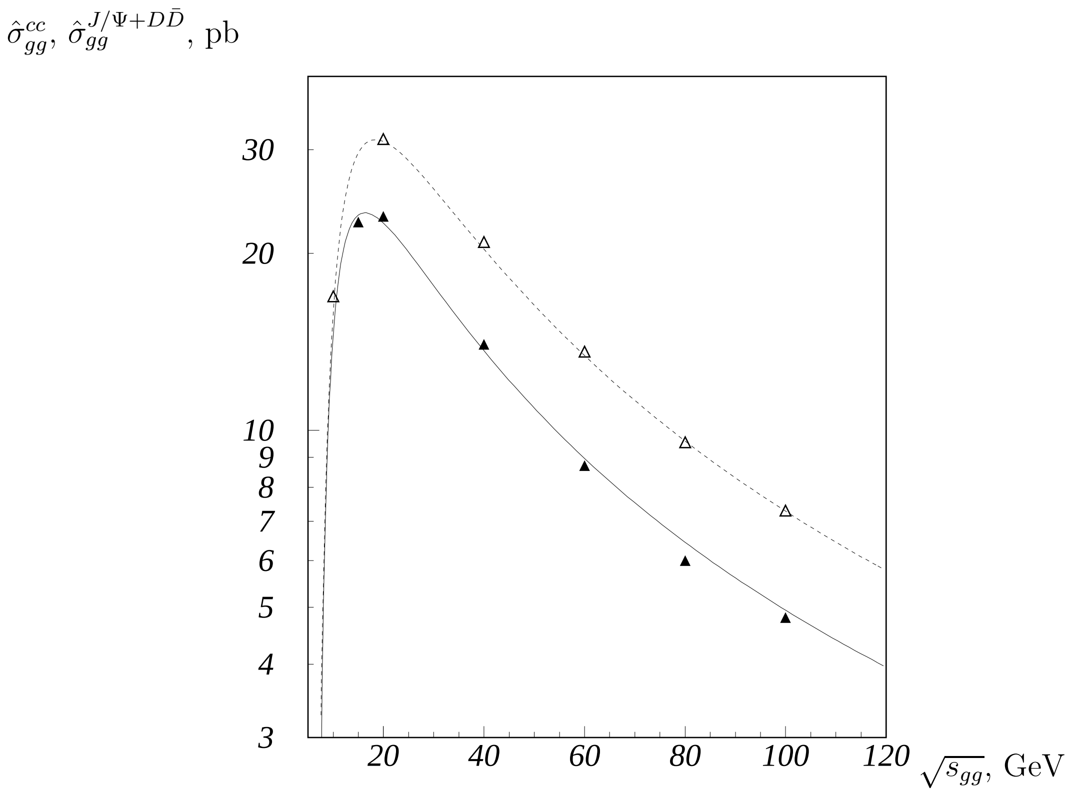

Figure 3.7. The total cross sections for the gluon-gluon production of cc diquark (▲) and $J/\Psi + D\bar{D}$ (△) in comparison with the approximations of (3.30) and (3.32) (solid and dashed lines, correspondingly).

As was mentioned, the production of J/Ψ in the subprocesses of $gg \to J/\Psi + c\bar{c}$ and $q\bar{q} \to J/\Psi + c\bar{c}$ is also calculated in this work. The numerical results of such consideration are shown in Figs. 3.7 and 3.8. The parameterization of these results versus the energy $\sqrt{\hat{s}}$ are presented below

$$\hat{\sigma}_{gg}^{J/\Psi} = 518. \left(1 - \frac{4m_c}{\sqrt{\hat{s}}}\right)^{3.0} \left(\frac{4m_c}{\sqrt{\hat{s}}}\right)^{1.45} \quad \text{pb}, \tag{3.32}$$

$$\hat{\sigma}_{q\bar{q}}^{J/\Psi} = 699. \left(1 - \frac{4m_c}{\sqrt{\hat{s}}}\right)^{1.9} \left(\frac{4m_c}{\sqrt{\hat{s}}}\right)^{2.97} \quad \text{pb}. \tag{3.33}$$

As well as for the associated $B_c + b\bar{c}$ and $\Xi_{cc} + \bar{c}\bar{c}$, we see the following regularity for the partonic production of $J/\psi + c\bar{c}$: the fragmentation regime occures at $p_T > 25 - 30$ GeV. This fact can be

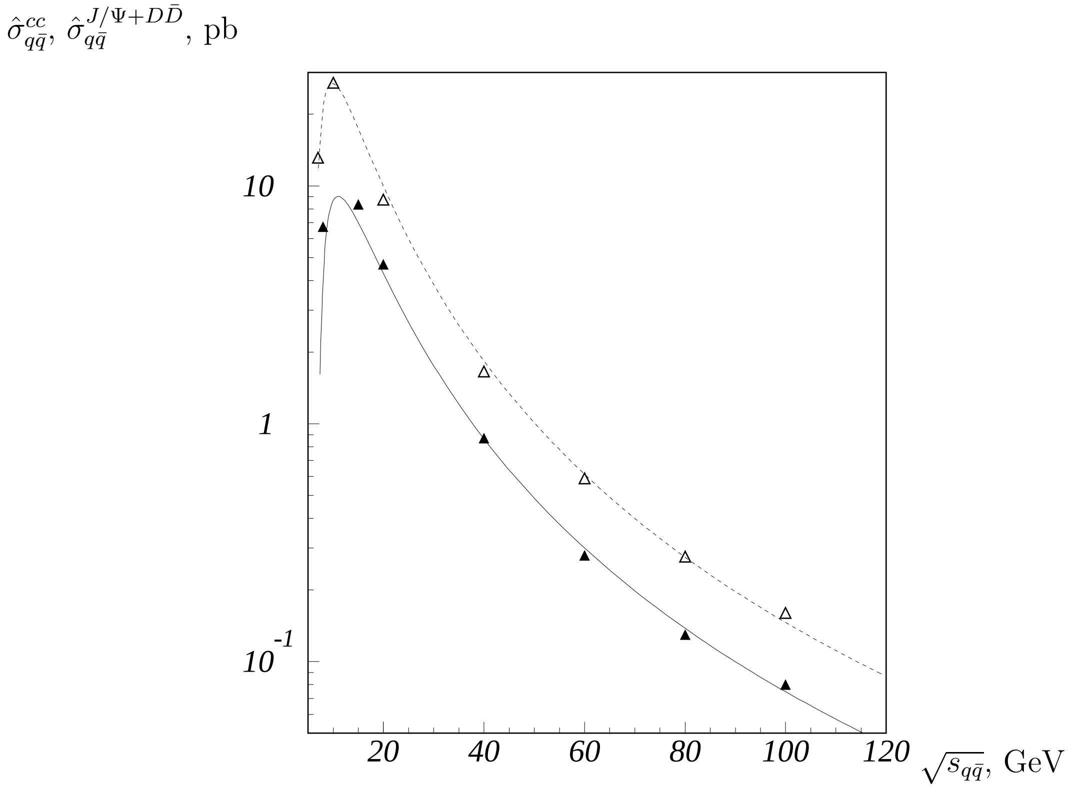

Figure 3.8. The total cross sections for the gluon-gluon production of cc diquark (▲) and $J/\Psi + D\bar{D}$ (△) in comparison with the approximations of (3.31) and (3.33) (solid and dashed lines, correspondingly).

certainly observed in Fig. 3.9 for the differential cross section of $gg \to J/\psi + c\bar{c}$ at $\sqrt{\hat{s}} = 100$ GeV. Thus, for the associated production of $J/\psi + c\bar{c}$ and $\Xi_{cc} + \bar{c}\bar{c}$ the fragmentation works at $p_T \gg m_c$.

The above parametrizations quite accurately reconstruct the results of precise calculations at $\sqrt{\hat{s}} < 150$ GeV, and that is why they can be used for the approximate estimation of total hadronic production cross section for the cc-diquark and J/Ψ by means of their convolution with the partonic distributions

$$\sigma = \sum_{i,j} \int dx_1 dx_2 f_{i/A}(x_1, \mu) f_{j/B}(x_2, \mu) \hat{\sigma}, \tag{3.34}$$

where $f_{i/A}(x, \mu)$ is the distribution of i-kind parton in the A-hadron. The parton distributions used for the proton are the CTEQ4 parameterizations [69], and those of used for the π^--meson are the Hpdf ones [70]. In both cases the virtuality scale is fixed at 10 GeV. As for the choice of fixed scale in the structure functions, this is caused by the fact that the cross section of subprocesses is integrated in the region of low $\hat{s}$ close to the fixed scale, so that the account of "running" scale weakly changes the estimate of Ξ_{cc}-baryon yield in comparison with the mentioned uncertainty of diquark model[14]. The cross sections convoluted with the gluon and quark luminosities are presented in Fig. 3.10 for both the cc-diquark and J/Ψ production in $\pi^- p$ and pp collisions.

As we can see in these figures, the cross section of cc-diquark as well as the cross section of $J/\Psi + c\bar{c}$ are strongly suppressed at low energies in comparison with the values at the collider energies. The ratio for the cc-diquark production and total charm production is $\sigma_{cc}/\sigma_{charm} \sim 10^{-4} - 10^{-3}$ in the collider experiments and $\sim 10^{-6} - 10^{-5}$ in the fixed target experiments. The

[14]We have found the scale-dependent variation to be at the level of $\delta\sigma/\sigma \sim 10\%$.

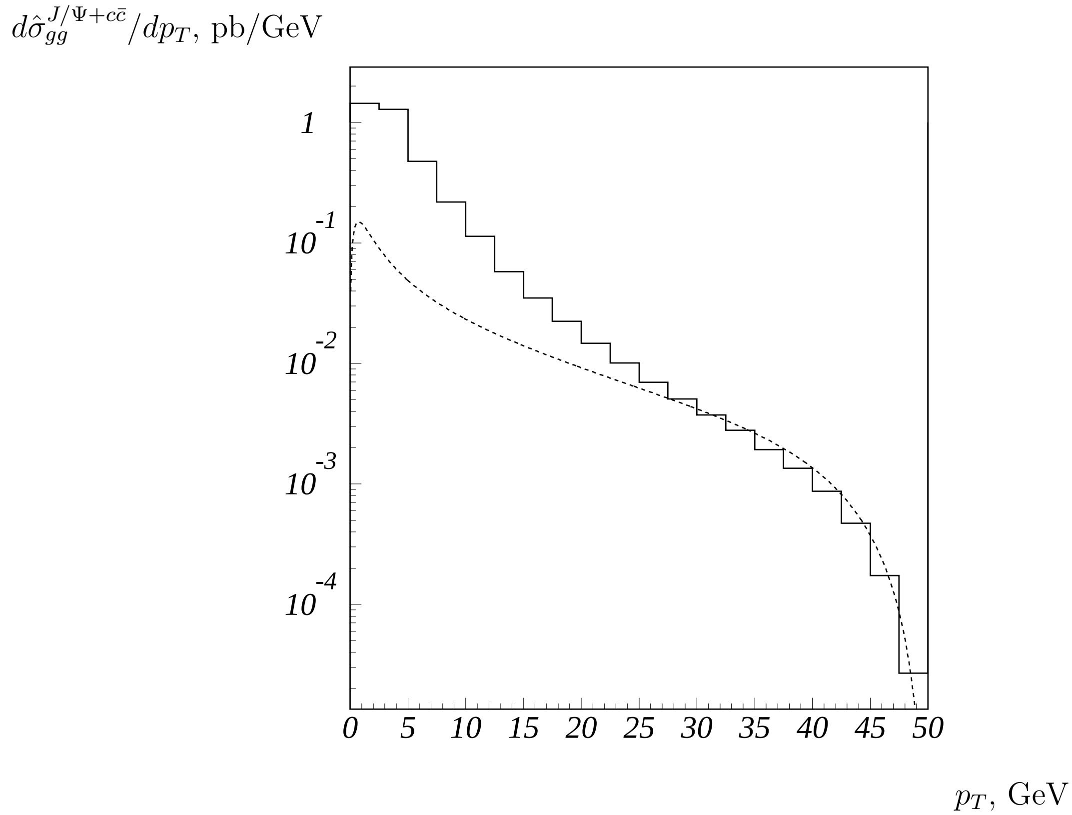

Figure 3.9. The differential cross section for the associated production of $J/\Psi + c\bar{c}$ in the gluon-gluon subprocess at 100 GeV (solid histogram) in comparison with the prediction of fragmentation model (dashed curve), correspondingly.

same situation is observed for the hadronic $J/\Psi + D\bar{D}$ production.

The distributions for the ccq-baryon and $J/\Psi + D\bar{D}$ production are shown in [71] for the $\pi^- p$-interaction at 35 GeV and for the pp-interaction at 40 GeV, correspondingly. The rapidity distributions point to the central character of ccq-baryon production as well as $J/\Psi + D\bar{D}$. The p_T-distributions of these processes are alike to each other also[15]. The form of differential distributions leads to the conclusion that the process of $J/\Psi + D\bar{D}$ production can be used to normalize the estimate of ccq-baryon yield, wherein the following additional uncertainties appear:

1. the unknown value of $|R_{(cc)}(0)|^2$,

2. uncertainties related with the hadronization of cc-diquark.

We see from the given estimates that in the experiments with the number of charmed events at the level about 10^6 (for example, in the E781 experiment, where $\sqrt{s} = 35$ GeV), we have to expect about one event with the doubly charmed baryon. The situation is more promising and pleasant for the pp-interaction at 800 GeV (HERA-B). The considered processes yield about 10^5 $\Xi_{cc}^{(*)}$-baryons and a close number of $J/\Psi + D\bar{D}$ in the experiment specialized for the detection of about 10^8 events with the b-quarks.

[15] We assume that at the given energies the cc-diquark has no fragmentational transition into the baryon, but it catches up the light quark from the quark-antiquark pair sea.

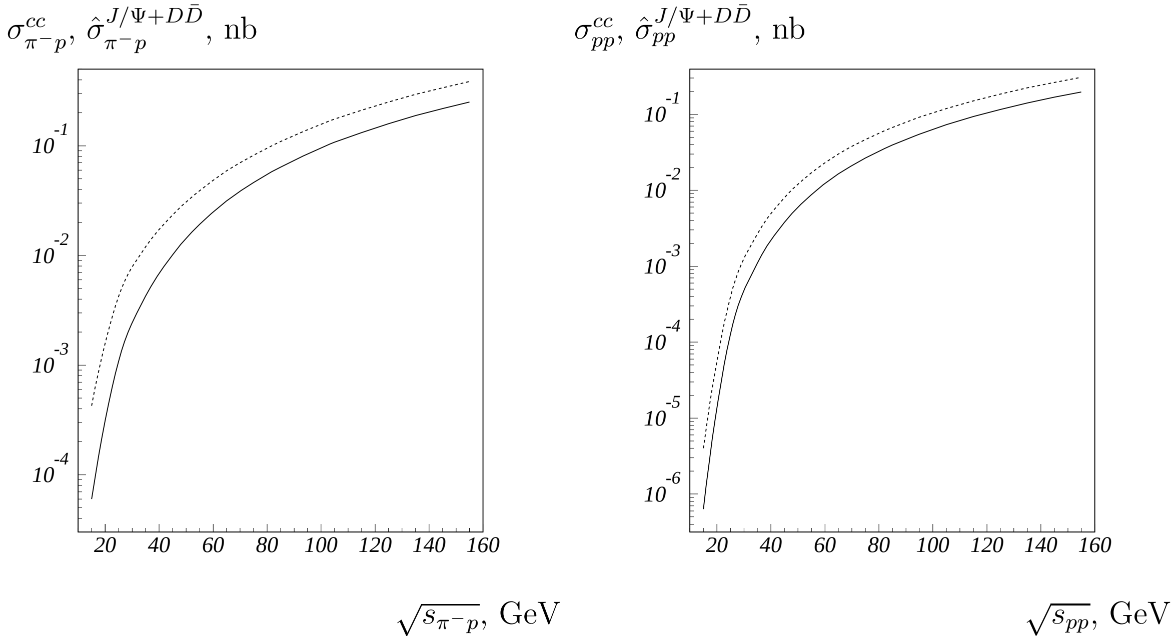

Figure 3.10. The total cross sections for the production of cc-diquark and $J/\Psi + D\bar{D}$ (solid and dashed curves, correspondingly) in $\pi^- p$ and pp collisions.

3.3.3. Production of ccq-baryon at colliders

As one can see in the previous Section, the observation of $\Xi_{cc}^{(*)}$-baryons presents a rather difficult problem in the experiments specialized for the study of charmed particles. As a rule, such experiments are carried out at fixed targets, so that the effective value of subprocess energy is strongly decreased. So, the relative contribution of doubly charmed baryons into the total charm yield is of the order of $10^{-6} - 10^{-5}$. The production of ccq-baryons at colliders with large p_T is more effective. In this case the cross section is determined by the region of quark-antiquark and gluon-gluonic energy, where the threshold effect becomes negligible and the partonic luminosities are quite large at $x \sim M/\sqrt{s}$. So, the suppression factor with respect to the single production of $c\bar{c}$-pairs is much less and it is in the range of $10^{-4} - 10^{-3}$.

In [71] the p_T-distributions for $\Xi_{cc}^{(*)}$ and J/Ψ associated with D and $\bar{D}$ are shown at the energies of Tevatron and LHC with the rapidity cut $|y| < 1$.

One can easily understand that the presented $\Xi_{cc}^{(*)}$ cross sections are the upper estimates for the real cross sections because of the possible dissociation of heavy diquark into the DD-pair.

Further, even if the cc-diquark being the color object, transforms into the baryon with the unit probability, one has to introduce the fragmentation function describing the hadronization of diquark into the baryon at quite large p_T values. The simplest form of this function can be chosen by the analogy with that for the heavy quark

$$D(z) \sim \frac{1}{z} \frac{1}{(M^2 - \frac{m_{cc}^2}{z} - \frac{m_q^2}{1-z})^2}, \tag{3.35}$$

where M is the mass of baryon $\Xi_{cc}^{(*)}$, m_{cc} is the mass of diquark, m_q is the mass of light quark (we suppose it to be equal to 300 MeV). This function practically repeats the form of fragmentation function if diquark into the baryon, that has been derived above in this chapter in the framework of

perturbative QCD. In [71] the p_T-distributions of doubly charmed baryon production are calculated with the use of (3.35). One has to mention, that in the leading order over the inverse heavy quark mass, the relative yield of Ξ_{cc} and Ξ^*_{cc} is determined by the simple counting rule for the spin states, and it equals $\sigma(\Xi_{cc}) : \sigma(\Xi^*_{cc}) = 1 : 2$. In this approach one does not take into account a possible difference between the fragmentation functions for the baryons with the different spins. The corresponding difference is observed in the perturbative fragmentation functions for the heavy mesons and quarkonia [51, 59].

3.3.4. Hadronic production of Ξ_{bc}

The total energy dependence of the gluonic production cross sections of Ξ'_{bc} ($\circ$) and $\Xi^{(*)}_{bc}$ ($\bullet$) baryons is shown in Fig. 3.11. For the sake of comparison, the predictions of the fragmentational mechanism for $\Xi^{(*)}_{bc}$ (solid line) and Ξ'_{bc} (dashed line) are also presented. One can see from the figure, that the fragmentational production mechanism assuming the validity of factorization in the cross section at $M^2/s \ll 1$ and at large transverse momenta in accordance with the formula

$$\frac{d\sigma_{gg\to\Xi'_{bc}(\Xi^{(*)}_{bc})\bar b\bar c}}{dz} = \sigma_{gg\to b\bar b}\cdot D_{b\to\Xi'_{bc}(\Xi^{(*)}_{bc})}(z), \tag{3.36}$$

where $z = 2|\vec{P}|/\sqrt{s}$, does not work at low gluon energies, where it overestimates the cross section, because of the incorrect evaluation of the phase space, and it is also not valid at large energies, where the predictions of the fragmentational mechanism are essentially less that the exact results.

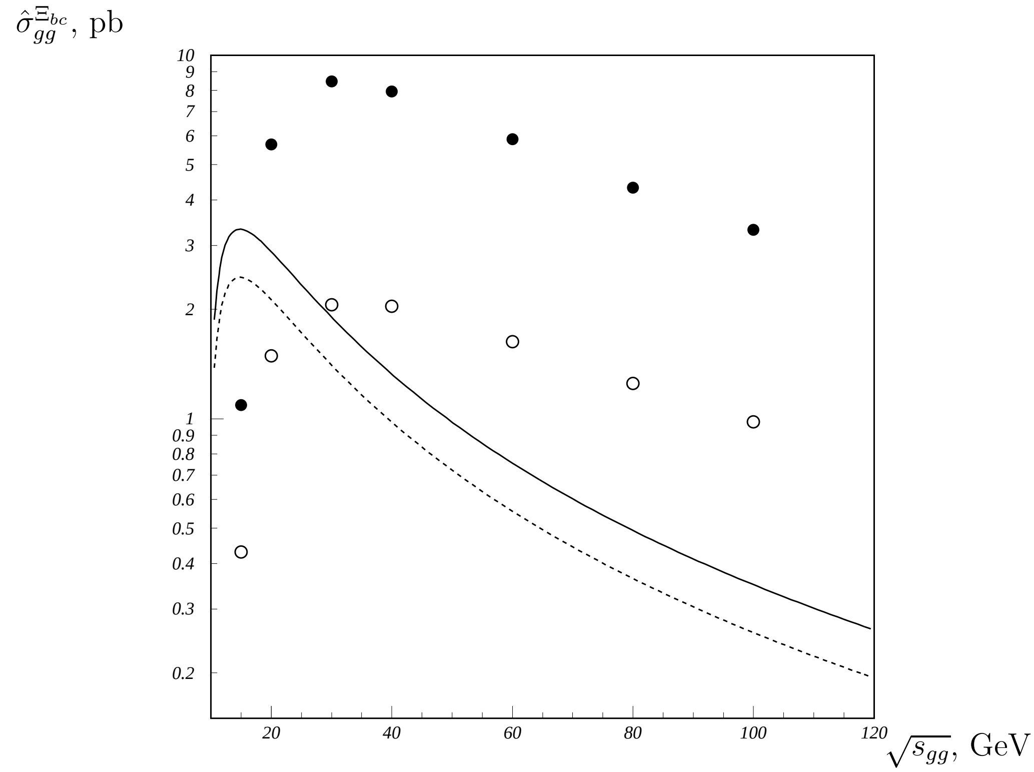

Figure 3.11. The gluonic production cross sections of Ξ'_{bc} ($\circ$) and $\Xi^{(*)}_{bc}$ ($\bullet$) in comparison with the predictions of the fragmentational mechanism for Ξ'_{bc} (dashed line) and $\Xi^{(*)}_{bc}$ (solid line).

So, the fragmentational values underestimate the $\Xi_{bc}^{(*)}$ and Ξ'_{bc} cross sections by 10 and 3 times, respectively, at $\sqrt{\hat{s}} = 100$ GeV. When the fragmentational predictions give the ratio $\sigma_{\Xi_{bc}^{(*)}}/\sigma_{\Xi'_{bc}} \simeq 1.4$, the exact perturbative calculations result in $\sigma_{\Xi_{bc}^{(*)}}/\sigma_{\Xi'_{bc}} \simeq 3.5$ even at $\sqrt{\hat{s}} = 100$ GeV.

The agreement with the fragmentational production at $\sqrt{\hat{s}} = 100$ GeV takes place at large transverse momenta of the baryon, as one can see from the distributions over p_T for the $\Xi_{bc}^{(*)}$ and Ξ'_{bc} production, shown in Fig. 3.12. Note, that in contrast to the doubly heavy baryon production, the exact perturbative calculations of the gluonic production of $B_c(B_c^*)$ mesons with $p_T > 35$ GeV at $\sqrt{\hat{s}} = 100$ GeV agree with the fragmentational predictions. For the baryon production, a visible deviation is observed up to the largest values of p_T, though it is evident that at larger energies of gluon-gluon subprocess the region of justified use of fragmentation regime will become wider in the direction of large transverse momenta.

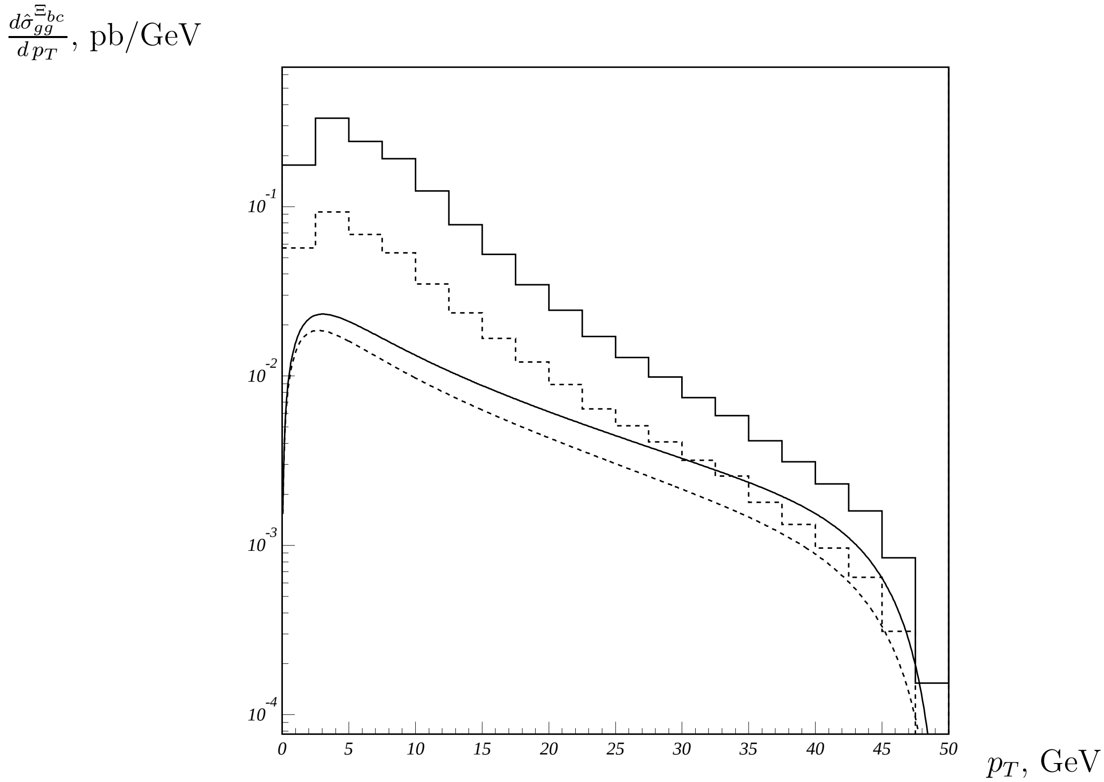

Figure 3.12. The distributions over the transverse momentum in the gluonic production of $\Xi'_{bc}(\Xi_{bc}^{(*)})$ in comparison with the fragmentation result at the interaction energy of 100 GeV. The solid line corresponds to the production of $\Xi_{bc}^{(*)}$, the dashed curve represents the yield of Ξ'_{bc}, while the complete results and fragmentation estimates are given by the histogram and smooth curves, correspondingly.

The differential cross section $d\sigma/dp_T$ of the Ξ'_{bc} and $\Xi_{bc}^{(*)}$ production in $p\bar{p}$ interactions at $\sqrt{s} = 1.8$ TeV is presented in [71] in comparison with the fragmentational predictions. So, we can also conclude that the fragmentation approach gives rather rough estimate for the yield and momentum distribution of Ξ_{bc} baryons.

At the chosen values of parameters and with the account for the cuts over the transverse momentum and rapidity of the baryons ($p_T > 5$ GeV and $|y| < 1$), the production cross section of the $1S$-wave bcq-baryons and its anti-particles is evaluated as $\sigma_{bcq} \simeq 1$ nb (without cuts the value of σ_{bcq} is about two times greater). After the expected end of Run Ib at Tevatron with the integral

luminosity $100 \div 150$ pb^{-1}, one has the yields of $1.0 \div 1.5 \cdot 10^5$ of bcq-baryons.

3.3.5. Pair production

At the energies of fixed target experiments the luminosity of parton subprocesses with the valence quarks is not suppressed in comparison with the luminosity of gluon-gluon collisions in the region of large invariant masses. In this way a significant fraction of total cross section for the production of baryons with two heavy quarks is given by the pair production of such baryons. The total and differential cross sections for the pair production were considered in [56], wherein the contributions of scalar and axial-vector diquarks were taken into account. So, the expression for the total cross section of scalar pairs versus the square of total energy s has the form

$$\sigma_{ss} = \frac{8\pi^3}{81 s^3}\,|\Psi_s(0)|^4 \left(1-\frac{4M^2}{s}\right)^{3/2} \left(\frac{16}{3}(\tilde{f}^{[1]}_{ss})^2 + \frac{11}{20}\left(1-\frac{4M^2}{s}\right)(\tilde{f}^{[2]}_{ss})^2\right), \tag{3.37}$$

where $\Psi_s(0)$ is the diquark wave function at the origin, while the formfactors are determined by the functions

$$\tilde{f}^{[1]}_{ss} = M\alpha_s^2 \left[\left(\frac{1}{m_1^2}+\frac{1}{m_2^2}\right) - \frac{2M^2}{s}\left(\frac{m_2}{m_1^3}+\frac{m_1}{m_2^3}\right)\right], \tag{3.38}$$

$$\tilde{f}^{[2]}_{ss} = \frac{M^5}{m_1^3 m_2^3}\alpha_s^2, \tag{3.39}$$

so that $m_{1,2}$ are the heavy quark masses, and $M = m_1 + m_2$.

Numerical estimates show that the pair production of vector diquarks dominates, so that it gives about 10% of single doubly heavy baryon production in the parton subprocess.

3.3.6. Discussion

On the basic of perturbative calculations for the hard production of doubly charmed diquark fragmentating in the baryon we have shown that the observation of doubly charmed baryons is a difficult problem, because the ratio of $\sigma(\Xi_{cc}^{(*)})/\sigma(charm)$ for these baryons and charmed particles yields the value of $10^{-6} - 10^{-3}$ depending on the process energy. The suppression of doubly charmed baryon yield at low energies of fixed target experiments is explained by the threshold effect. As one can

Table 3.2. The total cross section of doubly charmed baryon production at different facilities.

facility	HERA-B	E781	Tevatron	LHC
total cross section, nb/nucleon	$2 \cdot 10^{-3}$	$4.6 \cdot 10^{-3}$	12	122

see in Table 3.2, the low value of cross section for the production of doubly charmed baryons in the fixed target experiments allows us to expect about 10^5 events with the production of these baryons at HERA-B. Practically the same number of events at $p_T > 5$ GeV and $|y| < 1$ is expected at Tevatron with the integrated luminosity of 100 pb^{-1}. The large luminosity and large interaction energy allow one to increase the yield of the doubly charmed baryon by 10^4 times at LHC.

Taking into account the increase of luminosity of collider at FNAL, we can believe that the experimental search for the events with the baryons Ξ_{bc} and Ξ_{cc} is the actual task.

Under conditions of large yield of the doubly charmed baryons, the problem of their registration is under challenge.

First of all it is interesting to estimate the lifetimes of the lightest states of Ξ_{cc}^{++} and Ξ_{cc}^{+}. The simple study of quark diagrams shows that in the decay of Ξ_{cc}^{++}-baryon the Pauli interference for the decay products of charmed quark and valence quark in the initial state takes place as well as in the case of D^+-meson decay. In the decay of Ξ_{cc}^{+} the exchange by the W-boson between the valence quarks plays an important role as well as in the annihilation of D^0. Therefore we suppose that the mentioned mechanisms[16] give the same ratio for the both baryon and D-meson lifetimes

$$\tau(\Xi_{cc}^{+}) \approx 0.4 \cdot \tau(\Xi_{cc}^{++}).$$

Further, the presence of two charmed quark in the initial state results in following expressions:

$$\begin{aligned}
\tau(\Xi_{cc}^{++}) &\approx \frac{1}{2}\tau(D^+) \simeq 0.53 \text{ ps},\\
\tau(\Xi_{cc}^{+}) &\approx \frac{1}{2}\tau(D^0) \simeq 0.21 \text{ ps}.
\end{aligned}$$

Following the analogy with the decays of baryon containing a single heavy quark, we can expect the decay modes

$$\begin{aligned}
&\mathrm{BR}(\Xi_{cc}^{++} \to K^{0(*)}\Sigma_c^{++(*)}) \approx \mathrm{BR}(\Xi_{cc}^{+} \to K^{0(*)}(\Sigma_c^{+(*)} + \Lambda_c^{+})) \approx\\
&\approx \mathrm{BR}(\Lambda_c \to K^{0(*)}p) \simeq 4 \cdot 10^{-2}.
\end{aligned}$$

One can observe $4 \cdot 10^3$ events in these decay modes at HERA-B and Tevatron without taking into account a detection efficiency. One has to expect the yield of $4 \cdot 10^7$ such decays at LHC. Among other decay modes, the $\Xi_{cc}^{++} \to \pi^+\Xi_c^+$ and $\Xi_{cc}^{+} \to \pi^+\Xi_c^0$ processes taking place with branching ratios of about 1%, can be essential.

The excited Ξ_{cc}^* states always decay into Ξ_{cc} by the emission of γ-quantum, so that the branching fraction of transition is equal to 100%, since the emission of π-meson is impossible in the Ξ_{cc}^* decay because of the small value of splitting between the ground state and the excited one, in contrast to the charmed meson decay.

In conclusion we mention another possibility to increase the yield of doubly charmed baryons in fixed target experiments. In the model of intrinsic charm [72] one assumes, that the nonperturbative admixture of exotic hybrid state $|c\bar{c}uud\rangle$ presents in the proton along with the ordinary state $|uud\rangle$ including three light valence quarks.The probability P_{ic} of $|c\bar{c}uud\rangle$-state is suppressed at the level of 1%. The valence charmed quark from that state can recombine with the charmed quark produced in the hard partonic process with the $c\bar{c}$-pair production. The energy dependence for such doubly charmed baryon production repeats one for the single charmed quark production in the framework of pQCD up to the factor of exotic state suppression and the factor of fusion of two charmed quarks into the diquark, $K \sim 0.1$. This mechanism has no threshold of four quark state production in contrast to the discussed perturbative one. Therefore at low energies of fixed target experiments, where the threshold suppression of perturbative mechanism is strong, the model of intrinsic charm

[16] We present quite naive estimates for the lifetimes, while a strict consideration of operator product expansion for the inclusive decays of doubly heavy baryons is addressed in Chapter 4.

would yield the dominant contribution into the $\Xi_{cc}^{(*)}$ production. So, the number of events in this model would be increased by three orders of magnitude, and ratio of $\Xi_{cc}^{(*)}$ and charmed particle yields would equal $\sigma(\Xi_{cc}^{(*)})/\sigma(\text{charm}) \sim 10^{-3}$. At high energies the perturbative production is comparable with the intrinsic charm contribution. We have to note that the $|c\bar{c}c\bar{c}uud\rangle$-state suppressed at the level of $3 \cdot 10^{-4}$, also could increase the doubly charmed baryon production at low energies of hadron-hadron collisions.

Thus, the observation of Ξ_{cc}^{*}-baryons in hadronic interactions is a quite realistic problem, whose solution opens new possibilities to research the heavy quark interactions. The observation of $\Xi_{cc}^{(*)}$-baryons at fixed target experiments [73] would allow one to investigate the contributions of different mechanisms in the doubly charmed baryon production, as the contribution of the perturbative mechanism and that of the intrinsic charm, which strongly increases the yield of these baryons.

Chapter 4. Lifetimes and decays of $\Xi_{QQ'}$ baryons

In the framework of Operator Product Expansion (OPE) over the inverse powers of heavy quark mass there is a definite scheme for consistent calculations of QCD effects, that was developed for the evaluation of various characteristics in decays of hadrons containing the heavy quarks [9, 10, 74]. The consideration based on the predictions of this approach allows one to extract the parameters assigned to the electroweak interactions of heavy quarks in the background of dynamics with strong interactions of quarks and gluons, which form the hadrons observed in the experimental measurements. The accuracy of QCD description in the sector of heavy quarks has an important significance in the search for subtle effects such as the violation of CP-invariance, deviations from the predictions of Standard Model as well as in clarifying the mechanisms for contributions of virtual corrections caused by a "new" physics operating on characteristic energy scales of TeV range. Therefore, the study of OPE over the inverse powers of heavy quark mass is quite an informative problem of interest serving a manyfold attention. An importamt challenge is a complex investigation of system containing the heavy quarks with the analysis and comparison of various characteristics such as I) the convergency of expansion in both the inverse mass and the QCD coupling constant, II) relative and absolute values of various contributions and their dependence on the quark contents of systems, III) qualitative conclusions on the role of some mechanisms [75], and IV) uncertainties of numerical estimates.

The efficiency of approach under discussion was convincingly shown in the description of weak decays for the hadrons having a single heavy quark, carried out in the framework of the heavy quark effective theory (HQET) [9], in the annihilation and radiative decays of heavy quarkonia $Q\bar{Q}$, using the framework of nonrelativistic QCD (NRQCD) [10], and in the weak decays of long-lived heavy quarkonium with mixed heavy flavours[17] B_c^+ [74]. We emphasize that the experimental data on the weak decays of doubly heavy hadrons could be able to bring a significant improvement of numerical accuracy in the parameters entering the description of systems with the heavy quarks. The advantage is caused by the presence of an additional small parameter in the NRQCD expansion in contrast to HQET. This parameter is a relative velocity of heavy quark motion v. Moreover, there is an essential variability of characteristics assigned to the bound states with the heavy quarks, so that these changes of properties allow us to investigate the dependence of OPE on the nonperturbative parameters, which can be evaluated, for example, in the potential approach.

The baryons with two heavy quarks, $QQ'q$, provide a new insight in the description of systems containing the heavy quarks. For these baryons we can apply a method based on the combined HQET-NRQCD techniques [9,10,74], if we use the quark-diquark factorization for the bound states. The expansion in the inverse powers of heavy quark mass for the heavy diquark QQ' is a straightforward generalization of these techniques in the mesonic decays [10,74] with the difference that we deal with the color anti-triplet system of heavy quarks with the appropriate account of interaction with the light quark instead of the color singlet systems. The HQET methods have to reliably work for describing the interaction of local diquark with the light quark.

In this chapter, we present the consistent calculation of lifetimes for the doubly heavy baryons Ξ_{bc}^+, Ξ_{bc}^0, Ξ_{cc}^+ and Ξ_{cc}^{++}. Taking into account necessary generalizations to the case of hadrons with two heavy quarks and other corrections in the description of inclusive decays for the baryons, we follow the general consideration of heavy hadron decays in [6, 74], where the decays of the hadrons with a single heavy quark and the doubly heavy B_c meson were discussed. The justified

[17] The first experimental observation of B_c-meson has been reported by the CDF Collaboration [5], while a review on the theoretical status of B_c is given in [4].

basis for such calculations is the optical theorem for the inclusive decay width combined with the Operator Product Expansion (OPE) for the transition currents in accordance with the consequent nonrelativistic expansion of hadronic matrix elements derived in OPE. Using OPE at the first step, we exploit the fact, that, due to the presence of heavy quarks in the initial state, the energy release in the decay of both quarks is large enough in comparison with the binding energy in the state. Thus, we can use the expansion over the ratio of these scales. Technically, this step repeats an analogous procedure for the inclusive decays of heavy-light mesons as it was reviewed in [76]. Exploring the nonrelativistic expansion of hadronic matrix elements at the second step, we use the approximation of nonrelativistic QCD [10], which allows one to reduce the evaluation of matrix elements for the full QCD operators, corresponding to the interaction of heavy quarks inside the diquark, to the expansion in powers of $\frac{p_c}{m_c}$, where $p_c = m_c v_c \sim 1\ GeV$ is a typical momentum of the heavy quark inside the baryon. The same procedure for the matrix elements, determined by the strong interaction of heavy quarks with the light quark, leads to the expansion in powers of $\frac{\Lambda_{QCD}}{m_c}$.

We note, that keeping just the leading term in the OPE, the inclusive widths are determined by the mechanism of spectator decays involving free quarks, wherein the corrections in the perturbative QCD are taken into account. The introduction of subleading terms in the expansion over the inverse heavy quarks mass[18] allows one to take into account the corrections due to the quark confinement inside the hadron. In this way, an essential role is played by the following nonperturbative characteristics: the motion of heavy quark inside the hadron and the corresponding time dilation in the hadron rest frame with respect to the quark rest frame, and the influence of the chromo-magnetic interaction of the quarks. The important ingredient of such corrections in the baryons with two heavy quarks is the presence of a compact heavy diquark. The next peculiarity of baryons with two heavy quarks is the significant numerical influence on the lifetimes by the quark contents of hadrons, since in the third order of inverse heavy quark mass, $1/m_Q^3$, the four-quark correlations in the total width are enforced in the effective lagrangian due to the two-particle phase space in the intermediate state (see discussion in [6]). In this situation, we have to add the effects of the Pauli interference between the products of heavy quark decays and the quarks in the initial state as well as the weak scattering involving the quarks composing the hadron. Through such terms we introduce the corrections involving the masses of light and strange quarks in the framework of nonrelativistic models with the constituent quarks. We include the corrections to the effective weak lagrangian due to the evolution of Wilson coefficients from the scale of the order of heavy quark mass to the energy, characterizing the binding of quarks inside the hadron.

Following the picture given above, we describe the general scheme for the construction of OPE for the total widths of baryons with two heavy quarks with account of the corrections to the spectator widths in Section 4.1. In Section 4.2, the procedure for the estimation of nonperturbative matrix elements over the states of doubly heavy baryons is considered for the operators of nonrelativistic heavy quarks. Section 4.3 is devoted to the numerical evaluation of lifetimes for the baryons with two heavy quarks and their partial decay rates, as well as to the discussion of underlying uncertainties. We conclude the chapter by summarizing our results.

4.1. Operator Product Expansion for the heavy baryons

In this section we describe the general scheme for the construction of OPE for the total widths of baryons containing two heavy quarks.

[18]It was shown in [76] that the leading order correction in $1/m_Q$ is absent and the corrections begin with $1/m_Q^2$.

Now let us start the description of our approach for the calculation of lifetimes for the doubly charmed baryons. The optical theorem, taking into account the integral quark-hadron duality, allows us to relate the total decay width of the heavy hadron Γ with the imaginary part of its forward scattering amplitude. This relationship, applied to the $\Xi_{cc}^{\diamond}$-baryon total decay width $\Gamma_{\Xi_{cc}^{\diamond}}$, can be written down as

$$\Gamma_{\Xi_{cc}^{\diamond}} = \frac{1}{2M_{\Xi_{cc}^{\diamond}}}\langle \Xi_{cc}^{\diamond}|\mathcal{T}|\Xi_{cc}^{\diamond}\rangle, \tag{4.1}$$

where the state $\Xi_{cc}^{\diamond}$ in (4.1) has the ordinary relativistic normalization

$$\langle \Xi_{cc}^{\diamond}|\Xi_{cc}^{\diamond}\rangle = 2EV,$$

and the transition operator $\mathcal{T}$ is determined by the expression

$$\mathcal{T} = \Im m \int d^4x \;\{\mathrm{T} H_{eff}(x) H_{eff}(0)\}, \tag{4.2}$$

where H_{eff} is the standard effective hamiltonian describing the low energy weak interactions of initial quarks with the decay products. For the transition of c-quark into the u-quark and the quarks $q_{1,2}$ with the charge $-1/3$, the lagrangian has the form

$$H_{eff} = \frac{G_F}{2\sqrt{2}} V_{uq_1} V^*_{cq_1}[C_+(\mu)O_+ + C_-(\mu)O_-] + \mathrm{h.c.} \tag{4.3}$$

where V is the matrix of mixing between the charged currents, and

$$O_{\pm} = [\bar{q}_{1\alpha}\gamma_\nu(1-\gamma_5)c_\beta][\bar{u}_\gamma\gamma^\nu(1-\gamma_5)q_{2\delta}](\delta_{\alpha\beta}\delta_{\gamma\delta} \pm \delta_{\alpha\delta}\delta_{\gamma\beta}),$$

so tat the indices α, β denote the color states of quarks, and

$$C_+ = \left[\frac{\alpha_s(M_W)}{\alpha_s(\mu)}\right]^{\frac{6}{33-2n_f}}, \quad C_- = \left[\frac{\alpha_s(M_W)}{\alpha_s(\mu)}\right]^{\frac{-12}{33-2n_f}},$$

n_f is the number of flavors.

Assuming that the energy release in the heavy quark decay is large, we can perform the operator product expansion for the transition operator $\mathcal{T}$ in (4.3). In this way we find a series of local operators with increasing dimensions over the energy scale, wherein the contributions to Γ are suppressed by the increasing inverse powers of the heavy quark masses. This formalism was already applied to calculate the total decay rates for the hadrons, containing a single heavy quark (see refs. [76] and [6, 77]). Here we would like to stress that the expansion, applied in this paper, is simultaneously in the powers of the inverse heavy quark mass and the relative velocity of heavy quarks inside the hadron. Thus, this fact points to the difference from the description of both the heavy-light mesons (the expansion in powers of $\frac{\Lambda_{QCD}}{m_c}$) and the heavy-heavy mesons [74] (the expansion in powers of relative velocity of heavy quarks inside the hadron, where one can apply the scaling rules of nonrelativistic QCD [10]).

In this work we will extend this approach to the treatment of baryons containing two heavy quarks. The operator product expansion applied has the following form:

$$\mathcal{T} = C_1(\mu)\bar{c}c + \frac{1}{m_c^2}C_2(\mu)\bar{c}g\sigma_{\mu\nu}G^{\mu\nu}c + \frac{1}{m_c^3}O(1). \tag{4.4}$$

The leading contribution in OPE is determined by the operator $\bar{c}c$, corresponding to the spectator decays of c-quarks. The use of the equation of motion for the heavy quark fields allows one to eliminate some redundant operators, so that no operators of dimension four contribute. There is a single operator of dimension five, $Q_{GQ} = \bar{Q}g\sigma_{\mu\nu}G^{\mu\nu}Q$. As we will show below, significant contributions come from the operators of dimension six $Q_{2Q2q} = \bar{Q}\Gamma q\bar{q}\Gamma' Q$, representing the effects of Pauli interference and weak scattering for Ξ_{cc}^{++} and Ξ_{cc}^{+}, correspondingly, which are enforced by the two-particle phase space. Furthermore, there are also other operators of dimension six $Q_{61Q} = \bar{Q}\sigma_{\mu\nu}\gamma_l D^\mu G^{\nu l}Q$ and $Q_{62Q} = \bar{Q}D_\mu G^{\mu\nu}\Gamma_\nu Q$. In what follows, we neglect the corresponding contributions for the latter two operators, since they are suppressed by the mentioned smallness of three-particle phase space, so that the expansion is certainly complete up to the second order of $\frac{1}{m}$, only. The reason for the restriction of dimension-six operators is twofold. First, the operators Q_{61Q} and Q_{62Q} do not contribute to the lifetime difference between the doubly charmed baryons under consideration, since they are independent of the quark contents of hadrons. Second, the four-quark operators are enhanced in comparison with the quark-gluon terms with the same dimension because the two-particle phase space integrated in the calculation of coefficients in front of Pauli interference and weak scattering has the additional factor of $16\pi^2$ in contrast to the the three-particle phase space expressed in the units of heavy quark mass as it occurs for the coefficients of operators Q_{61Q} and Q_{62Q}, so that they are suppressed.

Further, the different contributions to OPE are given by the following:

$$\mathcal{T}_{\Xi_{cc}^{++}} = \mathcal{T}_{35c} + \mathcal{T}_{6,PI},$$

$$\mathcal{T}_{\Xi_{cc}^{+}} = \mathcal{T}_{35c} + \mathcal{T}_{6,WS},$$

where the first terms account for the operators of dimension three O_{3Q} and five O_{GQ}, the second terms correspond to the effects of Pauli interference and weak scattering. The explicit formulae for these contributions have the following form:

$$\mathcal{T}_{35c} = 2(\Gamma_{c,spec}\bar{c}c - \frac{\Gamma_{0c}}{m_c^2}[(2 + K_{0c})P_1 + K_{2c}P_2]O_{Gc}), \tag{4.5}$$

where $\Gamma_{0c} = \frac{G_F^2 m_c^2}{192\pi^3}$ and $K_{0c} = C_-^2 + 2C_+^2$, $K_{2c} = 2(C_+^2 - C_-^2)$. This expression has been derived in [78] (see also [79]), and it is also discussed in [76]. The phase space factors P_i [76, 80] have the form

$$P_1 = (1 - y)^4, \quad P_2 = (1 - y)^3$$

where $y = \frac{m_s^2}{m_c^2}$. $\Gamma_{c,spec}$ denotes the contribution to the total decay width of the free decay for one of the two c-quarks, which is explicitly expressed in [81].

Quite cumbersome expressions for the contributions of Pauli interference and weak scattering terms in the inclusive widths are given in Appendix III.

In the numerical estimates for the evolution of coefficients C_+ and C_-, we have taken into account the threshold effects, connected to the b-quark, as well as the threshold effects, related to the c-quark mass in the Pauli interference and weak scattering.

In the expressions for $C_\pm$ the scale μ is approximately equal to m_c. For the Pauli interference and weak scattering, this scale in the factor k is chosen in the way to obtain an agreement between the experimental differences of lifetimes for the Λ_c, Ξ_c^+ and Ξ_c^0-baryons and the theoretical predictions based on the effects mentioned above. This problem is discussed below. Anyway, the choice of these

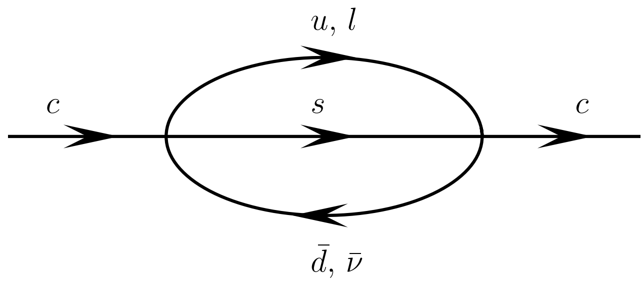

Figure 4.1. The spectator contribution into the total widths of doubly charmed baryons.

scales allows some variations, and a complete answer to this question requires calculations in the next order of perturbative theory.

The contribution of the leading operator $\bar{c}c$ corresponds to the imaginary part of the diagram in Fig. 4.1, as it stands in expression (4.5). The coefficient in front of $\bar{c}c$ can be obtained in the usual way by projection of contribution due to the diagram in Fig. 4.1 to the operator $\bar{c}c$. This coefficient is equivalent to the free quark decay rate, and it is known in the next-to-leading logarithmic approximation of QCD [82–86], including the strange quark mass effects in the final state [86]. To calculate the next-to-leading logarithmic effects, the Wilson coefficients in the effective weak lagrangian are required at the next-to-leading accuracy, and the single gluon exchange corrections to the diagram in Fig. 4.1 must be considered. In our numerical estimates we use the expression for Γ_{spec}, including the next-to-leading order corrections, s-quark mass effects in the final state, but we neglect the Cabibbo-suppressed decay channels for the c-quark. The bulky explicit expression for the spectator c-quark decay is placed in the Appendix of [81].

Similarly, the contribution by O_{GQ} is obtained, when an external gluon line is attached to the inner quark lines in Fig. 4.1 in all possible ways. The corresponding coefficients are known in the leading logarithmic approximation. Finally, the dimension six operators and their coefficients arise due to those contributions, wherein one of the internal u or $\bar{d}$ quark line is "cut". The resulting graphs are depicted in Figs. 4.2 and 4.3. These contributions correspond to the effects of Pauli interference and weak scattering. We have calculated the expressions for these effects with account for both the s-quark mass in the final state and the logarithmic renormalization of effective electroweak lagrangian at low energies. This effective lagrangian is shown in Appendix III.

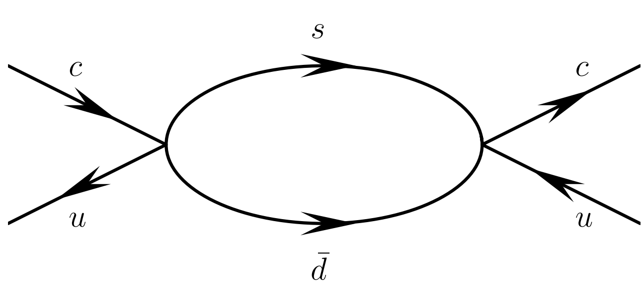

Figure 4.2. The Pauli interference of c-quark decay products with the valence quark in the initial state for the Ξ_{cc}^{++}-baryon.

To calculate the contribution of semileptonic modes to the total decay width of $\Xi_{cc}^{\diamond}$-baryons (we have taken into account the electron and muon decay modes only) we use the following expressions

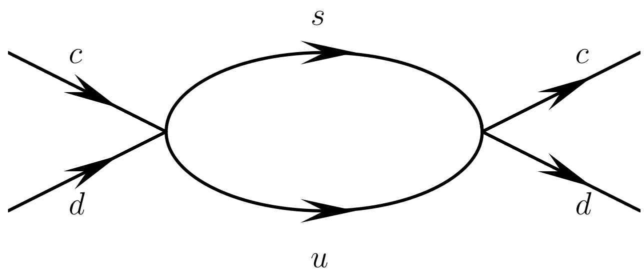

Figure 4.3. The weak scattering of the valence quarks in the initial state for the Ξ_{cc}^{+}-baryon.

[79] (see also [86]):

$$\begin{aligned}
\Gamma_{sl} &= 4\Gamma_c(\{1-8\rho+8\rho^3-\rho^4-12\rho^2\ln\rho\}+ \\
& E_c\{5-24\rho+24\rho^2-8\rho^3+3\rho^4-12\rho^2\ln\rho\}+ \\
& K_c\{-6+32\rho-24\rho^2-2\rho^4+24\rho^2\ln\rho\}+ \\
& G_c\{-2+16\rho-16\rho^3+2\rho^4+24\rho^2\ln\rho\}),
\end{aligned} \tag{4.6}$$

where $\Gamma_c = |V_{cs}|^2 G_F^2 \frac{m_c^5}{192\pi^3}$, $\rho = \frac{m_s^2}{m_c^2}$, $E_c = K_c + G_c$, while K_c and G_c are given by the expressions

$$\begin{aligned}
K_c &= -\langle \Xi_{cc}^{\diamond}(v)|\bar{c}_v \frac{(i\mathbf{D})^2}{2m_c^2} c_v|\Xi_{cc}^{\diamond}(v)\rangle, \\
G_c &= \langle \Xi_{cc}^{\diamond}(v)|\bar{c}_v \frac{gG_{\alpha\beta}\sigma^{\alpha\beta}}{4m_c^2} c_v|\Xi_{cc}^{\diamond}(v)\rangle,
\end{aligned} \tag{4.7}$$

where the spinor field c_v in the effective heavy quark theory is defined by the form

$$c(x) = e^{-im_c v\cdot x}\Big[1+\frac{iD_\mu\gamma^\mu}{2m_c}\Big]c_v(x), \tag{4.8}$$

where D_μ denotes the partial derivative over coordinates.

The analogous scheme for the calculation of inclusive widths $\Gamma_{\Xi_{bc}^{\diamond}}$ for the baryon $\Xi_{bc}^{\diamond}$, where $\diamond$ denotes the electric charge of the system, was developed in [87]. Then, for the total widths we have the expressions

$$\begin{aligned}
\mathcal{T}_{\Xi_{bc}^{+}} &= \mathcal{T}_{35b}+\mathcal{T}_{35c}+\mathcal{T}_{6,PI}^{(1)}+\mathcal{T}_{6,WS}^{(1)}, \\
\mathcal{T}_{\Xi_{bc}^{0}} &= \mathcal{T}_{35b}+\mathcal{T}_{35c}+\mathcal{T}_{6,PI}^{(2)}+\mathcal{T}_{6,WS}^{(2)},
\end{aligned}$$

where two initial terms denote the contributions into the decays of quark Q by the operators with the dimensions 3 and 5, and the forthcoming terms are the interference and scattering of constituents. Various contributions were explicitly presented in [87].

The calculation of both the Pauli interference effect for the products of heavy quark decays with the quarks in the initial state and the weak scattering of quarks composing the hadron, results in summming up the various channels of decays, so that

$$\mathcal{T}_{6,PI}^{(1)} = \mathcal{T}_{PI,u\bar{d}}^{c}+\mathcal{T}_{PI,s\bar{c}}^{b}+\mathcal{T}_{PI,d\bar{u}}^{b}+\sum_{l}\mathcal{T}_{PI,l\bar{\nu}_l}^{b},$$

$$\begin{aligned}
\mathcal{T}^{(2)}_{6,PI} &= \mathcal{T}^{b}_{PI,s\bar{c}} + \mathcal{T}^{b}_{PI,d\bar{u}} + \mathcal{T}'^{b}_{PI,d\bar{u}} + \sum_{l} \mathcal{T}^{b}_{PI,l\bar{\nu}_l}, \qquad (4.9)\\
\mathcal{T}^{(1)}_{6,WS} &= \mathcal{T}_{WS,bu} + \mathcal{T}_{WS,bc},\\
\mathcal{T}^{(2)}_{6,WS} &= \mathcal{T}_{WS,cd} + \mathcal{T}_{WS,bc}.
\end{aligned}$$

and

$$\begin{aligned}
\mathcal{T}^{b}_{PI,d\bar{u}} &= \mathcal{T}^{b}_{PI,s\bar{c}}\ (z_- \to 0),\\
\mathcal{T}^{b}_{PI,e\bar{\nu}_e} &= \mathcal{T}^{b}_{PI,\mu\bar{\nu}_\mu} = \mathcal{T}^{b}_{PI,\tau\bar{\nu}_\tau}\ (z_\tau \to 0). \qquad (4.10)
\end{aligned}$$

Further, we have taken into account the known α_s-corrections to the semileptonic widths of quarks.

Thus, the calculation of lifetimes for the baryons with two heavy quarks is reduced to the problem of evaluating the matrix elements of operators, that is the subject of next section.

4.2. Hadronic matrix elements

According to the equation of motion for the heavy quarks, the matrix element of local operator $\bar{Q}^j Q^j$ can be expanded in the following series over the powers of $1/m_{Q^j}$, so that

$$\frac{\langle \Xi^{\diamond}_{QQ'}|\bar{Q}^j Q^j|\Xi^{\diamond}_{QQ'}\rangle}{2M} = 1 - \frac{\langle \Xi^{\diamond}_{QQ'}|\bar{Q}^j[(i\boldsymbol{D})^2 - (\frac{i}{2}\sigma G)]Q^j|\Xi^{\diamond}_{QQ'}\rangle}{4Mm^2_{Q^j}} + O(\frac{1}{Mm^3_{Q^j}}). \qquad (4.11)$$

Thus, we have to make numerical estimates for the following list of operators:

$$\begin{aligned}
&\bar{Q}^j(i\boldsymbol{D})^2 Q^j, \quad (\frac{i}{2})\bar{Q}^j\sigma G Q^j, \quad \bar{Q}^j\gamma_\alpha(1-\gamma_5)Q^j\bar{q}\gamma^\alpha(1-\gamma_5)q,\\
&\bar{Q}^j\gamma_\alpha\gamma_5 Q^j\bar{q}\gamma^\alpha(1-\gamma_5)q, \quad \bar{Q}^j\gamma_\alpha\gamma_5 Q^j\bar{Q}^k\gamma^\alpha(1-\gamma_5)Q^k, \qquad (4.12)\\
&\bar{Q}^j\gamma_\alpha(1-\gamma_5)Q^j\bar{Q}^k\gamma^\alpha(1-\gamma_5)Q^k.
\end{aligned}$$

The first operator corresponds to the time dilation connected to the motion of heavy quarks inside the hadron. The second is related to the spin interaction of heavy quarks with the chromo-magnetic field of light quark and the other heavy quark. The further operators are the four-quark operators representing the effects of Pauli interference and weak scattering.

Next, following the general methods of effective theories, we introduce the effective filed Ψ_Q, which represents the nonrelativistic spinor of heavy quark, so that we integrate out the contributions with the virtualities μ in the range $m_Q > \mu > m_Q v_Q$ in the framework of perturbative QCD, while the nonperturbative effects in the matrix elements could be expressed in terms of effective nonrelativistic fields. So, we have

$$\begin{aligned}
\bar{Q}Q &= \Psi^\dagger_Q\Psi_Q - \frac{1}{2m^2_Q}\Psi^\dagger_Q(i\boldsymbol{D})^2\Psi_Q + \frac{3}{8m^4_Q}\Psi^\dagger_Q(i\boldsymbol{D})^4\Psi_Q -\\
&\quad \frac{1}{2m^2_Q}\Psi^\dagger_Q g\boldsymbol{\sigma}\boldsymbol{B}\Psi_Q - \frac{1}{4m^3_Q}\Psi^\dagger_Q(\boldsymbol{D}g\boldsymbol{E})\Psi_Q + ... \qquad (4.13)\\
\bar{Q}g\sigma_{\mu\nu}G^{\mu\nu}Q &= -2\Psi^\dagger_Q g\boldsymbol{\sigma}\boldsymbol{B}\Psi_Q - \frac{1}{m_Q}\Psi^\dagger_Q(\boldsymbol{D}g\boldsymbol{E})\Psi_Q + ... \qquad (4.14)
\end{aligned}$$

In these expressions we have omitted the term $\Psi_Q^\dagger \boldsymbol{\sigma}(g\boldsymbol{E}\times\boldsymbol{D})\Psi_Q$, corresponding to the spin-orbital interaction, because it vanishes in the ground states of baryons under consideration. For the normalization we take

$$\int d^3x \Psi_Q^\dagger \Psi_Q = \int d^3x Q^\dagger Q. \tag{4.15}$$

Then for Q determined by

$$Q \equiv e^{-imt}\begin{pmatrix}\phi\\ \chi\end{pmatrix} \tag{4.16}$$

we have

$$\Psi_c = \left(1+\frac{(i\boldsymbol{D})^2}{8m_c^2}\right)\phi. \tag{4.17}$$

Let us emphasize the difference in the descriptions of interactions between the heavy quarks and light quark and interactions between two heavy quarks. As we have mentioned, there is an additional small parameter in the heavy subsystem. It is the relative velocity of quark motion, that introduces the energetic scale $m_Q v$. Therefore, for example, the Darvin term $(\boldsymbol{D}\boldsymbol{E})$ in the heavy subsystem turns out to be the same order of magnitude in comparison with the chromo-magnetic term $(\boldsymbol{\sigma}\boldsymbol{B})$ in the expansion over the inverse power of heavy quark mass, since it has the same degree of smallness in v. This fact becomes clear if we explore the scaling relations derived in the nonrelativistic QCD [10]

$$\Psi_Q \sim (m_Q v_Q)^{\frac{3}{2}},\quad |\boldsymbol{D}| \sim m_Q v_Q,\quad g|\boldsymbol{E}| \sim m_Q^2 v_Q^3,\quad g|\boldsymbol{B}| \sim m_Q^2 v_Q^4,\quad g \sim v_Q^{\frac{1}{2}}.$$

For the interaction of heavy quark with the light one there is no small parameter defined by the relative velocity, so that the Darvin term is suppressed by the factor of $k/m_Q \sim \Lambda_{QCD}/m_Q$.

4.2.1. Baryons Ξ_{cc}^+ and Ξ_{cc}^{++}

Let us now start the calculation of matrix elements with the use of potential models for the bound states of hadrons. While estimating the matrix element value of the kinetic energy, we note, that the heavy quark kinetic energy consists of two parts: the kinetic energy of the heavy quark motion inside the diquark and the kinetic energy related to the diquark motion inside the hadron. According to the phenomenology of meson potential models, in the range of average distances between the quarks: 0.1 fm $< r <$ 1 fm, the average kinetic energy of quarks is constant and independent of both the quark flavors, composing the meson, and the quantum numbers, describing the excitations of the ground state. Therefore, we determine $T = m_d v_d^2/2 + m_l v_l^2/2$ as the average kinetic energy of diquark and light quark with $m_l = m_q$, and $T/2 = m_{c1}v_{c1}^2/2 + m_{c2}v_{c2}^2/2$ as the average kinetic energy of heavy quarks inside the diquark (the coefficient 1/2 takes into account the anti-symmetry of color wave function for the diquark). Finally, we have the following expression for the matrix element of the heavy quark kinetic energy:

$$\frac{\langle \Xi_{cc}^\diamond|\Psi_c^\dagger (i\mathbf{D})^2\Psi_c|\Xi_{cc}^\diamond\rangle}{2M_{\Xi_{cc}^\diamond} m_c^2} \simeq v_c^2 \simeq \frac{m_l T}{2m_c^2 + m_c m_l} + \frac{T}{2}. \tag{4.18}$$

We use the value $T \simeq 0.4$ GeV, which results in $v_c^2 = 0.146$, where the dominant contribution comes from the motion of heavy quarks inside the diquark.

Now we would like to estimate the matrix element of chromo-magnetic operator, corresponding to the interaction of heavy quarks with the chromo-magnetic field of the light quark. For this

purpose, we will use the following definitions: $O_{mag} = \sum_{i=1}^{2} \frac{g_s}{4m_c} \bar{c}^i \sigma_{\mu\nu} G^{\mu\nu} c^i$ and $O_{mag} \sim \lambda(j(j+1) - s_d(s_d+1) - s_l(s_l+1))$, where s_d is the diquark spin (as was noticed by the authors earlier, there is only the vector state of the cc-diquark in the ground state of such baryons), s_l is the light quark spin and j is the total spin of the baryon. Since both c-quarks additively contribute to the total decay width of baryons, we can use the diquark picture and substitute for the sum of c-quark spins the diquark spin. This leads to the parametrization for O_{mag}, as it is given above, and, moreover, it allows us to relate the value of the matrix element for this operator to the mass difference between the excited and ground state of baryons

$$O_{mag} = -\frac{2}{3}(M_{\Xi_{cc}^{*\diamond}} - M_{\Xi_{cc}^{\diamond}}). \tag{4.19}$$

The account for the interaction of heavy quarks inside the diquark leads to the following expressions for the chromo-magnetic and Darwin terms:

$$\frac{\langle \Xi_{cc}^{\diamond} | \Psi_c^{\dagger} g \boldsymbol{\sigma} \cdot \mathbf{B} \Psi_c | \Xi_{cc}^{\diamond} \rangle}{2M_{\Xi_{cc}^{\diamond}}} = \frac{2}{9} g^2 \frac{|\Psi^d(0)|^2}{m_c}, \tag{4.20}$$

$$\frac{\langle \Xi_{cc}^{\diamond} | \Psi_c^{\dagger} (\mathbf{D} \cdot g\mathbf{E}) \Psi_c | \Xi_{cc}^{\diamond} \rangle}{2M_{\Xi_{cc}^{\diamond}}} = \frac{2}{3} g^2 |\Psi^d(0)|^2, \tag{4.21}$$

where $\Psi^d(0)$ is the diquark wave function at the origin. Collecting the results given above, we find the matrix elements of dominant operators determining the spectator decay

$$\begin{aligned} \frac{\langle \Xi_{cc}^{\diamond} | \bar{c}c | \Xi_{cc}^{\diamond} \rangle}{2M_{\Xi_{cc}^{\diamond}}} &= 1 - \frac{1}{2} v_c^2 - \frac{1}{3} \frac{M_{\Xi_{cc}^{*\diamond}} - M_{\Xi_{cc}^{\diamond}}}{m_c} - \frac{g^2}{9m_c^3} |\Psi^d(0)|^2 - \frac{1}{6m_c^3} g^2 |\Psi^d(0)|^2 + \ldots \\ &\approx 1 - 0.074 - 0.004 - 0.003 - 0.005 \end{aligned} \tag{4.22}$$

We can see that the largest contribution to the decrease of the decay width comes from the time dilation, connected to the motion of heavy quarks inside the baryon. For the matrix element of the operator $\bar{c} g \sigma_{\mu\nu} G^{\mu\nu} c$, we get

$$\frac{\langle \Xi_{cc}^{\diamond} | \bar{c} g \sigma_{\mu\nu} G^{\mu\nu} c | \Xi_{cc}^{\diamond} \rangle}{2M_{\Xi_{cc}^{\diamond}} m_c^2} = -\frac{4}{3} \frac{(M_{\Xi_{cc}^{*\diamond}} - M_{\Xi_{cc}^{\diamond}})}{m_c} - \frac{4g^2}{9m_c^3} |\Psi^d(0)|^2 - \frac{g^2}{3m_c^3} |\Psi^d(0)|^2. \tag{4.23}$$

Now let us continue with the calculation of the matrix elements for the four-quark operators, corresponding to the effects of Pauli interference and weak scattering. The straightforward calculation in the framework of nonrelativistic QCD gives the following expressions for the operators $\langle \bar{c}\gamma_\mu(1-\gamma_5)c \cdot \bar{q}\gamma^\mu(1-\gamma_5)q \rangle$ and $\langle \bar{c}\gamma_\mu\gamma_5 c \cdot \bar{q}\gamma^\mu(1-\gamma_5)q \rangle$:

$$\langle \bar{c}\gamma_\mu(1-\gamma_5)c \cdot \bar{q}\gamma^\mu(1-\gamma_5)q \rangle = 2m_c V^{-1}(1 - 4S_c S_q), \tag{4.24}$$

$$\langle \bar{c}\gamma_\mu\gamma_5 c \cdot \bar{q}\gamma^\mu(1-\gamma_5)q \rangle = -4S_c S_q \cdot 2m_c V^{-1}, \tag{4.25}$$

where $V^{-1} = |\Psi_l(0)|^2$, $\Psi_l(0)$ is the light quark wave function at the origin of two c-quarks. We suppose that, for estimates, $|\Psi_l(0)|$ has the same value as that in the D-meson. So, we find

$$|\Psi_l(0)|^2 \approx \frac{f_D^2 m_D^2}{12m_c}. \tag{4.26}$$

We would like to note that in the numerical estimate the supposed value of light quark wave function obtained from the leptonic constant of charmed meson $f_D \approx 200$ MeV, is about two times less than the wave function calculated in Chapter 1 in the approximation of quark-diquark factorization. This is related with the fact that the leptonic constants of charmed hadrons have large corrections of both logarithmic form and powers of inverse charmed quark mass. So, the leptonic constant of D meson calculated in the potential models is two times greater than the value obtained in the QCD sum rules with account for the mentioned corrections. Therefore, we think that the supposed approximation for the wave function of light quark in the baryon is justified enough.

Then, again remembering that both c-quarks additively contribute to the total decay width and using the diquark picture, we can substitute for $S_{c_1} + S_{c_2}$ by S_d, where S_d is the diquark spin. Thus, for the matrix elements of operators under study we have the following expressions:

$$\langle \Xi_{cc}^{\diamond} | (\bar{c}\gamma_\mu (1-\gamma_5) c)(\bar{q}\gamma^\mu (1-\gamma_5) q) | \Xi_{cc}^{\diamond} \rangle = 12 m_c \cdot |\Psi_l(0)|^2, \tag{4.27}$$

$$\langle \Xi_{cc}^{\diamond} | (\bar{c}\gamma_\mu \gamma_5 c)(\bar{q}\gamma^\mu (1-\gamma_5) q) | \Xi_{cc}^{\diamond} \rangle = 8 m_c \cdot |\Psi_l(0)|^2. \tag{4.28}$$

The color anti-symmetry of the baryon wave function results in relations between the matrix elements of operators with the different sums over the color indices: $\langle \Xi_{cc}^{\diamond} | (\bar{c}_i T_\mu c_k)(\bar{q}_k \gamma^\mu (1-\gamma_5) q_i | \Xi_{cc}^{\diamond} \rangle = -\langle \Xi_{cc}^{\diamond} | (\bar{c} T_\mu c)(\bar{q}\gamma^\mu (1-\gamma_5) q | \Xi_{cc}^{\diamond} \rangle$, where T_μ is any spinor structure.

Thus, we have formally constructed the procedure for the evaluation of matrix elements obtained in the OPE of $\mathcal{T}$ for the baryon with two identical heavy quarks.

4.2.2. Baryons Ξ_{bc}^{+} and Ξ_{bc}^{0}

Considering the baryons with two heavy quarks of different flavors, we point out the modifications, which have to be introduced in the estimation of hadronic matrix elements for the quark operators determining the inclusive decay widths.

First of all, according to the quar-diquark factorization we get the following expressions for the kinetic terms:

$$\frac{\langle \Xi_{bc}^{\diamond} | \Psi_c^\dagger (i\boldsymbol{D})^2 \Psi_c | \Xi_{bc}^{\diamond} \rangle}{2 M_{\Xi_{bc}^{\diamond}} m_c^2} \simeq v_c^2 \simeq \frac{2 m_l T}{(m_l + m_b + m_c)(m_b + m_c)} + \frac{m_b T}{m_c (m_c + m_b)}. \tag{4.29}$$

$$\frac{\langle \Xi_{bc}^{\diamond} | \Psi_b^\dagger (i\boldsymbol{D})^2 \Psi_b | \Xi_{bc}^{\diamond} \rangle}{2 M_{\Xi_{bc}^{\diamond}} m_b^2} \simeq v_b^2 \simeq \frac{2 m_l T}{(m_l + m_b + m_c)(m_b + m_c)} + \frac{m_c T}{m_b (m_c + m_b)}. \tag{4.30}$$

Numerically, we assign $T \simeq 0.4$ GeV, which results in $v_c^2 = 0.195$ and $v_b^2 = 0.024$, where the dominant contribution is provided by the motion inside the diquark.

Let us define

$$\begin{aligned} O_{mag} &= \frac{g_s}{4 m_c} \bar{c} \sigma_{\mu\nu} G^{\mu\nu} c + \frac{g_s}{4 m_b} \bar{b} \sigma_{\mu\nu} G^{\mu\nu} b, & (4.31) \\ \langle O_{mag} \rangle &= \frac{\lambda}{m_c} (S_{cl}(S_{cl}+1) - S_c(S_c+1) - S_l(S_l+1)) + & (4.32) \\ & \quad \frac{\lambda}{m_b} (S_{bl}(S_{bl}+1) - S_b(S_b+1) - S_l(S_l+1)), & \end{aligned}$$

where $S_{bl} = S_b + S_l$, $S_{cl} = S_c + S_l$, S_b is the b-quark spin, S_c is that of c-quark, and S_l is the light quark spin. The operator under study is related to the hyperfine splitting in the baryon system

$$\begin{aligned} &\langle S_{bc} = 1, S = \frac{3}{2}|O_{mag}|S_{bc} = 1, S = \frac{3}{2}\rangle - \langle S_{bc} = 1, S = \frac{1}{2}|O_{mag}|S_{bc} = 1, S = \frac{1}{2}\rangle \\ &= \langle S_{bc} = 1, S = \frac{3}{2}|V_{hf}|S_{bc} = 1, S = \frac{3}{2}\rangle - \langle S_{bc} = 1, S = \frac{1}{2}|V_{hf}|S_{bc} = 1, S = \frac{1}{2}\rangle, \end{aligned} \tag{4.33}$$

so that S denotes the total spin of the system, and S_{bc} is the diquark spin. Further, the perturbative term, depending on the spins, is equal to

$$V_{hf} = \frac{8}{9}\alpha_s \frac{1}{m_l m_c}\mathbf{S}_l\mathbf{S}_c|R^{dl}(0)|^2 + \frac{8}{9}\alpha_s \frac{1}{m_l m_b}\mathbf{S}_l\mathbf{S}_b|R^{dl}(0)|^2, \tag{4.34}$$

where $R^{dl}(0)$ is the radial wave function at the origin of quark-diquark system. In contrast to the diquark system with the identical quarks, this operator is not diagonal in the basis of S and S_{bc}. To proceed, we use the change of basises

$$|S; S_{bc}\rangle = \sum_{S_{bl}} (-1)^{(S+S_l+S_c+S_b)}\sqrt{(2S_{bl}+1)(2S_{bc}+1)} \left\{ \begin{array}{ccc} \bar{S}_l & S_b & S_{bl} \\ S_c & S & S_{bc} \end{array} \right\} |S; S_{bl}\rangle \tag{4.35}$$

and

$$|S; S_{bc}\rangle = \sum_{S_{cl}} (-1)^{(S+S_l+S_c+S_b)}\sqrt{(2S_{cl}+1)(2S_{bc}+1)} \left\{ \begin{array}{ccc} \bar{S}_l & S_c & S_{cl} \\ S_b & S & S_{bc} \end{array} \right\} |S; S_{cl}\rangle \tag{4.36}$$

The result of substitutions gives

$$\lambda = \frac{4|R^{dl}(0)|^2\alpha_s}{9m_l}, \tag{4.37}$$

however, for the state with the zero spin of heavy diquark, which is considered in what follows, we have

$$\frac{\langle \Xi^{\diamond}_{bc}|O_{mag}|\Xi^{\diamond}_{bc}\rangle}{2M_{\Xi^{\diamond}_{bc}}} = 0 \tag{4.38}$$

The account of Darwin and chromo-magnetic terms results in

$$\frac{\langle \Xi^{\diamond}_{bc}|\Psi^{\dagger}_c g\boldsymbol{\sigma}\cdot\boldsymbol{B}\Psi_c|\Xi^{\diamond}_{bc}\rangle}{2M_{\Xi^{\diamond}_{bc}}} = -\frac{2}{3}g^2\frac{|\Psi^d(0)|^2}{m_b}, \tag{4.39}$$

$$\frac{\langle \Xi^{\diamond}_{bc}|\Psi^{\dagger}_c(\boldsymbol{D}\cdot g\boldsymbol{E})\Psi_c|\Xi^{\diamond}_{bc}\rangle}{2M_{\Xi^{\diamond}_{bc}}} = \frac{2}{3}g^2|\Psi^d(0)|^2. \tag{4.40}$$

where $\Psi^d(0)$ is the wave function at the origin of diquark. The analogous matrix elements[19] for the operators of beauty quarks can be written down by the permutation of heavy quark masses.

Combining the results above, we find

$$\begin{aligned} \frac{\langle \Xi^{\diamond}_{bc}|\bar{c}c|\Xi^{\diamond}_{bc}\rangle}{2M_{\Xi^{\diamond}_{bc}}} &= 1 - \frac{1}{2}v_c^2 + \frac{g^2}{3m_b m_c^2}|\Psi^d(0)|^2 - \frac{1}{6m_c^3}g^2|\Psi^d(0)|^2 + \ldots \\ &\approx 1 - 0.097 + 0.004 - 0.007\ldots \end{aligned} \tag{4.41}$$

[19]The obtained expressions differ from those for the B_c meson [74] because of the color structure of the state, providing the factor of $\frac{1}{2}$.

The dominant role in the corrections is played by the term, connected to the time dilation because of the quark motion inside the diquark. Next, for the operator $cg\sigma_{\mu\nu}G^{\mu\nu}c$ we have

$$\frac{\langle\Xi^{\diamond}_{bc}|\bar{c}g\sigma_{\mu\nu}G^{\mu\nu}c|\Xi^{\diamond}_{bc}\rangle}{2M_{\Xi^{\diamond}_{bc}}m_c^2} = \frac{4g^2}{3m_bm_c^2}|\Psi^d(0)|^2 - \frac{g^2}{3m_c^3}|\Psi^d(0)|^2 \approx 0.002. \tag{4.42}$$

The permutation of quark masses lead to the required expressions for the operators of $\bar{b}b$ and $\bar{b}g\sigma_{\mu\nu}G^{\mu\nu}b$.

Making use of (4.35) and (4.36) for the ground states of baryons results in

$$\begin{aligned}
\langle\Xi^{\diamond}_{bc}|(\bar{b}\gamma_\mu(1-\gamma_5)b)(\bar{c}\gamma^\mu(1-\gamma_5)c)|\Xi^{\diamond}_{bc}\rangle &= 8(m_b+m_c)\cdot|\Psi^d(0)|^2, &(4.43)\\
\langle\Xi^{\diamond}_{bc}|(\bar{b}\gamma_\mu\gamma_5 b)(\bar{c}\gamma^\mu(1-\gamma_5)c)|\Xi^{\diamond}_{bc}\rangle &= 6(m_b+m_c)\cdot|\Psi^d(0)|^2, &(4.44)\\
\langle\Xi^{\diamond}_{bc}|(\bar{b}\gamma_\mu(1-\gamma_5)b)(\bar{q}\gamma^\mu(1-\gamma_5)q)|\Xi^{\diamond}_{bc}\rangle &= 2(m_b+m_l)\cdot|\Psi^{dl}(0)|^2, &(4.45)\\
\langle\Xi^{\diamond}_{bc}|(\bar{b}\gamma_\mu\gamma_5 b)(\bar{q}\gamma^\mu(1-\gamma_5)q)|\Xi^{\diamond}_{bc}\rangle &= 0, &(4.46)\\
\langle\Xi^{\diamond}_{bc}|(\bar{c}\gamma_\mu(1-\gamma_5)c)(\bar{q}\gamma^\mu(1-\gamma_5)q)|\Xi^{\diamond}_{bc}\rangle &= 2(m_c+m_l)\cdot|\Psi^{dl}(0)|^2, &(4.47)\\
\langle\Xi^{\diamond}_{bc}|(\bar{c}\gamma_\mu\gamma_5 c)(\bar{q}\gamma^\mu(1-\gamma_5)q)|\Xi^{\diamond}_{bc}\rangle &= 0. &(4.48)
\end{aligned}$$

Thus, we can make numerucal estimates of inclusive decay widths.

4.3. Numerical estimates

Now we are ready to collect the contributions, described above, and to estimate the lifetimes of baryons Ξ_{cc}^{++} and Ξ_{cc}^{+}. For the beginning, we list the values of parameters, which we have used in our calculations, and give some comments on their choice.

$$\begin{array}{rclrclrcl}
m_c &=& 1.6\ \text{GeV}, & m_s &=& 0.45\ \text{GeV}, & |V_{cs}| &=& 0.9745,\\
M_{\Xi_{cc}^{++}} &=& 3.56\ \text{GeV}, & M_{\Xi_{cc}^{+}} &=& 3.56\ \text{GeV}, & M_{\Xi^{*\diamond}_{cc}} - M_{\Xi^{\diamond}_{cc}} &=& 0.1\ \text{GeV},\\
T &=& 0.4\ \text{GeV}, & |\Psi(0)| &=& 0.17\ \text{GeV}^{\frac{3}{2}}, & m_l &=& 0.30\ \text{GeV}.
\end{array} \tag{4.49}$$

For the parameters $M_{\Xi_{cc}^{++}}$, $M_{\Xi_{cc}^{+}}$ and $M_{\Xi^{*\diamond}_{cc}} - M_{\Xi^{\diamond}_{cc}}$ we use the mean values, given in the literature. Their evaluation has been also performed by the authors in the potential model for the doubly charmed baryons with the Buchmüller–Tye potential [26], and also in refs. [29, 32, 33, 38]. For f_D we use the value, given in [6, 76], and for T we take it from [88]. The mass m_c corresponds to the pole mass of the c-quark. For its determination we have used a fit of theoretical predictions for the lifetimes and semileptonic width of the D^0-meson from the experimental data. This choice of c-quark mass seems to effectively include unknown contributions of higher orders in perturbative QCD to the total decay width of baryons under consideration.

The renormalization scale μ is chosen in the following way: $\mu_1 = m_c$ in the estimate of Wilson coefficients C for the effective four-fermion weak lagrangian with the c-quarks at low energies and $\mu_2 = 1.2\ \ GeV$ for the Pauli interference and weak scattering. The latter value of renormalization scale has been obtained from the fit of theoretical predictions for the lifetimes differences of baryons Λ_c, Ξ_c^+, Ξ_c^0 over the experimental data. Here we would like to note, that the theoretical approximations used in [89] include the effect of logarithmic renormalization and do not take into account the mass effects, related to the s-quark in the final state. For the corresponding contributions to the decay widths of baryons with the different quark contents we have

$$\begin{aligned}
\triangle\Gamma_{nl}(\Lambda_c) &= c_d\langle O_d\rangle_{\Lambda_c} + c_u\langle O_u\rangle_{\Lambda_c},\\
\triangle\Gamma_{nl}(\Xi_c^+) &= c_s\langle O_s\rangle_{\Xi_c^+} + c_u\langle O_u\rangle_{\Xi_c^+},\\
\triangle\Gamma_{nl}(\Xi_c^0) &= c_d\langle O_d\rangle_{\Xi_c^0} + c_s\langle O_s\rangle_{\Xi_c^0},
\end{aligned} \tag{4.50}$$

where $\langle O_q \rangle_{X_c} = \langle X_c | O_q | X_c \rangle$, $O_q = (\bar{c}\gamma_\mu c)(\bar{q}\gamma^\mu q)$ with $q = u, d$ or s. The coefficients $c_q(\mu)$ are equal to

$$\begin{aligned}
c_d &= \frac{G_F^2 m_c^2}{4\pi}[C_+^2 + C_-^2 + \frac{1}{3}(4k^{\frac{1}{2}} - 1)(C_-^2 - C_+^2)], \\
c_u &= -\frac{G_F^2 m_c^2}{16\pi}[(C_+ + C_-)^2 + \frac{1}{3}(1 - 4k^{\frac{1}{2}})(5C_+^2 + C_-^2 - 6C_+C_-)], \\
c_s &= -\frac{G_F^2 m_c^2}{16\pi}[(C_+ - C_-)^2 + \frac{1}{3}(1 - 4k^{\frac{1}{2}})(5C_+^2 + C_-^2 + 6C_+C_-)].
\end{aligned} \tag{4.51}$$

We use the spin averaged value of the D-meson mass for the estimation of the effective light quark mass $m_l = \bar{\Lambda}$ as it stands below

$$m_D = m_c + \bar{\Lambda} + \frac{\mu_\pi^2}{2m_c} = m_c + m_l + \frac{T \cdot m_l}{m_c + m_l} = 1.98 \ \ \text{GeV}. \tag{4.52}$$

The s-quark mass can be written down in terms of m_l introduced above, so that

$$m_s = m_l + 0.15 \ \ \text{GeV}. \tag{4.53}$$

As we have already mentioned, the spectator decay width of c-quark $\Gamma_{c,spec}$ is known in the next-to-leading order of perturbative QCD [82–86]. The most complete calculation, including the mass effects, connected to the s-quark in the final state, is given in [86]. In the present work we have used these results for the calculation of the spectator contribution to the total decay width of doubly charmed baryons. In the calculation of the semileptonic decay width, we neglect the electron and muon masses in the final state. Moreover, we neglect the τ-lepton mode.

Now, let us proceed with the numerical analysis of contributions by the different decay modes into the total decay width. In Table 4.1 we have listed the results for the fixed values of parameters, described above. From this table one can see the significance of effects caused by both the Pauli interference and the weak scattering in the decays of doubly charmed baryons. The Pauli interference gives the negative correction about 63% for the Ξ_{cc}^{++}-baryons, and the weak scattering increases the total width by 61% for Ξ_{cc}^{+}. As we have already noted above, these effects take place separately in these baryons, and, thus, they enhance the difference of lifetimes.

Table 4.1. The contributions of different modes to the total decay width of doubly charmed baryons.

Mode or decay mechanism	Width, ps^{-1}	Contribution,% (Ξ_{cc}^{++})	Contribution,% (Ξ_{cc}^{+})
$c \to s\bar{d}u$	2.648	127	31
$c \to se^+\nu$	0.380	18	4.2
PI	-1.317	-63	–
WS	5.254	–	60.6
$\Gamma_{\Xi_{cc}^{++}}$	2.089	100	–
$\Gamma_{\Xi_{cc}^{+}}$	8.660	–	100

It is worth here to recall that the lifetime difference of D^+ and D^0-mesons is generally explained by the Pauli interference of c-quark decay products with the anti-quark in the initial state, while

in the current consideration, we see the dominant contribution of weak scattering. This could not be surprising, because under a more detailed consideration we will find, that the formula for the Pauli interference operator for the D-meson coincides with that for the weak scattering in the case of baryons, containing, at least, a single c-quark.

Finally, collecting the different contributions for the total lifetimes of doubly charmed baryons, we obtain the following values:

$$\tau_{\Xi_{cc}^{++}} = 0.48 \ \ \text{ps}, \quad \tau_{\Xi_{cc}^{+}} = 0.12 \ \ \text{ps}.$$

Note that the supposed exploration of fitting the data on the semileptonic decay width of D mesons, the difference between the widths of baryons containing the charmed quark as well the spectroscopic characteristics allow us to significantly decrease the variations of model parameters, i.e. the quark masses, the scale for the normalization of Wilson coefficients, and the wave function of light quark in the nonrelativistic model. In this way we decrease the uncertainties of theoretical predictions. The variation of charmed quark mass in the limits of $1.6-1.65$ GeV and difference between the masses of strange quark and light quark in the range $0.15-0.2$ GeV results in the following uncertainties for the lifetimes of baryons under consideration: $\delta\tau_{\Xi_{cc}^{++}} = 0.1$ ps, $\delta\tau_{\Xi_{cc}^{+}} = 0.01$ ps, whereas we see that the abolute values of uncertainties in the widths are equal to $\delta\Gamma_{\Xi_{cc}^{++}} = 0.4$ ps^{-1}, $\delta\Gamma_{\Xi_{cc}^{+}} = 0.9$ ps^{-1}. However, since the total width of Ξ_{cc}^{+} is significantly enhanced by the contribution of weak scattering for the constituent quarks, the relative uncertainty in the estimate of lifetimes for this baryon is much less, i.e. it is 10% in comparison with 20% for Ξ_{cc}^{++}).

Calculating the inclusive widths of decays for the Ξ_{bc}^{+} and Ξ_{bc}^{0} we have used

$$m_b = m_c + 3.5 \text{ GeV}, \tag{4.54}$$

in addition to the parameters given above. The baryon mass has been put to 7 GeV. For the wave function in the subsystem of diquark we have used the results of calculations in the nonrelativistic model with the potential by Buchmüller and Tye [13] corrected for the color structure of diquark, so that

$$\Psi^d(0) = 0.193 \ \ \text{GeV}^{3/2}.$$

Further, it is quite evident that the estimates of spectator widths of free heavy quarks do not depend on the system, wherein they are bound, so that we can exploit the results of the calculations performed earlier. We have chosen the quark masses to be the same as in [74], and we have put the corresponding values, presented in Table 4.2.

Table 4.2. The widths of inclusive spectator decays for the b and c quarks, in ps^{-1}.

mode	$b \to c\bar{u}d$	$b \to c\bar{c}s$	$b \to ce^+\nu$
Γ	0.310	0.137	0.075
mode	$b \to c\tau^+\nu$	$c \to s\bar{d}u$	$c \to se^-\bar{\nu}$
Γ	0.018	0.905	0.162

Then the procedure, described above with the shown parameters, leads to the lifetimes of the Ξ_{bc}^{+} and Ξ_{bc}^{0} baryons equal to

$$\tau_{\Xi_{bc}^{+}} = 0.33 \ \ \text{ps}, \quad \tau_{\Xi_{bc}^{0}} = 0.28 \ \ \text{ps}. \tag{4.55}$$

We can clearly see that the difference in the lifetimes caused by the decay processes with the Pauli interference and weak scattering is about 20 %. The relative contributions by various terms in the total widths of the baryons under consideration are presented in Table 4.3.

Table 4.3. The branching fractions (in %) of various modes in the inclusive decays of Ξ_{bc}^+ and Ξ_{bc}^0 baryons.

mode	Γ_b	Γ_c	Γ_{PI}	Γ_{WS}
Ξ_{bc}^+	20	37	23	20
Ξ_{bc}^0	17	31	21	31

Note that the contributions by the Pauli interference and weak scattering depending on the baryon contents, can be quite significant, up to $40 - 50\%$. The corrections due to the quark-gluon operators of dimension 5 are numerically very small. The most important corrections are those due to the operator of dimension 3, where the role of time dilation is essential for the heavy quarks in the hadron rest frame.

For the semileptonic decays, whose relative fractions are presented in Table 4.4, the largest corrections appear in the decays of b-quark because of the Pauli interference, so that the corresponding widths practically increase twice. This leads to the result that the semileptonic widths of b and c quarks in the electron mode are equal to each other, whereas for the spectator decays, the width of the charmed quark is twice that of b.

Table 4.4. The branching ratios for the inclusive semileptonic widths of Ξ_{bc}^+ and Ξ_{bc}^0, in %.

mode	$\Gamma^{e\nu}$	$\Gamma_b^{e\nu}$	$\Gamma_b^{\tau\nu}$
Ξ_{bc}^+	5.0	4.9	2.3
Ξ_{bc}^0	4.2	4.1	1.9

As for the sign of terms, caused by the Pauli interference, it is basically determined by the leading contribution, coming from the interference for the charmed quark of the initial state with the charmed quark from the b-quark decay. In this way, the anti-symmetric color structure of baryon wave function leads to the positive sign for the Pauli interference.

Finally, concerning the uncertainties of the estimates presented, we note that they are mainly related to the following:

1) the spectator width of charmed quark, where the error can reach 50 %, reflecting the agreement of theoretical evaluation with the lifetimes of charmed hadrons, so that for the baryons under consideration this term produces an uncertainty of $\delta\Gamma/\Gamma \approx 10\%$,

2) the effects of Pauli interference in the decays of beauty quark and in its weak scattering off the charmed quark from the initial state, wherein we use the nonrelativistic wave function, which, being model-dependent, can lead to an error estimated close to 30%, producing an uncertainty of $\delta\Gamma/\Gamma \approx 15\%$ in the total widths.

Thus, we estimate, that the uncertainty in the predictions of total widths for the Ξ_{bc}^+ and Ξ_{bc}^0 baryons is about 20%.

4.3.1. Parametric dependence of results

Despite of that we have already pointed out what is the expected accuracy of predictions on the inclusive widths and lifetimes of baryons with two heavy quarks, let us consider this important problem in a more detail way.

First of all, we have to investigate the dependence of widths on the masses of heavy quarks composing the diquarks in the baryons. Indeed, the spectator decay widths of heavy quarks are determined by the fifth power of masses, while the dominant corrections due to the Pauli interference and weak scattering of constituents depend as the third degree of the heavy quark masses. In this way, a natural challenge is the justification of quark-hadron duality with the OPE of quark operators, that leads to zero term of correction in the first inverse power of heavy quark mass, $1/m_Q$. Such the nullification follows from the theorem by Adomolo–Gato claiming that the introduction of term, which is proportional to λ and breaking a symmetry of lagrangian, leads to corrections of second order, i.e. λ^2, in observable quantities, so that under the introduction of $1/m_Q$ in the lagrangian of heavy quarks, the decay widths of heavy hadrons do not contain contributions linear in $1/m_Q$, if we suppose the quark-hadron duality. In the connection with the problem on the lifetime of Λ_b baryon, whose total width measured experimentally is about 20% greater than the widths of B mesons, that is in a contradiction with the predictions of heavy quark theory [90], authors of [91] suggested the hypothesis on a strong violation of quark-hadron duality, i.e. on the possibility of significant contribution by terms linear in $1/m_Q$ into the inclusive decay widths of heavy hadrons. This assumption, in fact, implies that the heavy quark effective mass determining the contribution of leading term, can vary with the mass and contents of hadron. So, considering a more large system of Λ_b baryon with the light diquark, wherein the string tension is twice less than the tension in the meson, we have to introduce a larger effective mass of heavy quark, since it is determined at a smaller energetic scale[20]. Then, the total width of Λ_b increases. Such the approach is not acceptable in the OPE with the quark-hadron duality, which operates with the heavy quark mass being the same for all kinds of hadrons, otherwise the corrections linear in $1/m_Q$ could appear in the widths. However, the hypothesis of the strong violation of quark-hadron duality is practically removed by the experimental measurement of B_c meson lifetime. Indeed, the author of [92] following the ideology of [91] predicted the value of lifetime $\tau[B_c] \approx 1.3 - 1.5$ ps, since the quarks inside the heavy quarkonium are strongly bound, and their effective masses as well as the admissible phase space of the final state in decays decrease. This fact leads to a significant suppression of decay widths for both $\bar{b}$ and c quarks in the B_c meson. The experimental measurement yields $\tau[B_c] = 0.48 \pm 0.19$ ps, which is in a good agreement with the estimates in the framework of OPE [74, 93], QCD sum rules [94] and potential models [4]. Thus, at present we can claim that the OPE with the quark-hadron duality is a correct tool in the calculations of inclusive decay widths and lifetimes for the hadrons with heavy quarks.

As we have mentioned, the estimates presented in the above sections have been done under the assumption that the chosen value of charmed quark mass results in quite the precise agreement of theoretical prediction on the semileptonic widths of D mesons with the experimental values. Behind our paper [81] the analogous calculations were performed in ref. [95], whose authors supposed essentially lower value for the charmed quark mass $m_c = 1.35$ GeV, which certainly resulted in that the semileptonic widths of charmed mesons could not be satisfactorily described in the same approach. Such the preference was probably caused by the following: first, a low value of current mass of c quark is usually obtained in the QCD sum rules for the charmonium, and, second, the

[20] The cloud of virtual gluons and quarks has a larger size in the heavy baryon than in the heavy-light meson.

description of inclusive decay widths of D mesons is generally considered to be quite qualitative, but quantitative, because the charmed quark mass is not large enough, therefore, the convergency of series in $1/m_Q$ could be slow[21]. Such the suggestions do lead to just a qualitative predictions on the lifetimes of baryons with two heavy quarks in [95], so, their estimates are twice or thrice different from the values obtained in previous sections. Indeed, the significant decrease of leading term because of the variation of charmed quark mass leads to that the negative contribution of Pauli interference strongly diminishes the total widths of Ξ_{cc}^{++}, so that the lifetime is essentially overestimated.

Another important source of uncertainty in the estimates of widths for the decays of charmed quark is the mass of strange quark. Indeed, since the charmed quark has the mass about 1.5 GeV, the phase space of its decay strongly depends on the chosen mass of strange quark: the current mass about $150 - 200$ MeV or the constituent mass close to the mass of K meson. In previous estimates we have supposed that the suppression of phase space is determined by the constituent mass. The problem on the dependence of inclusive decay widths on the masses of heavy and strange quarks was studied in detail in ref. [96] under the consideration of B_c meson lifetime, where the uncertainty caused by the modelling the wave function is low[22], and, moreover, the corresponding contribution of weak annihilation (corrections in the second order of $1/m_Q$) is small (about 10%).

Accepting (4.54), that follows from the analysis of data on the decays of B mesons, authors of [96] get the estimates presented in Table 4.5. In this table we see that, first, the low value of charmed quark mass supposed in [95] gives quite an overestimating value for the lifetime of B_c. Second, the choice of current mass for the strange quark is slightly preferable, since it leads to the value of B_c lifetime, which well agrees with the central value of experimental inteval, though the uncertainty of data allows the description with the constituent mass of strange quark, too. It is worth to note that this analysis yields the quark mass values consistent with the choice of charmed quark mass under the semileptonic decay widths of D mesons.

Table 4.5. The lifetime of B_c meson and contributions of spectator widths as well as of corrections caused by the Pauli interference (PI) and weak annihilation (WA) at different values of quark masses.

Parameters, GeV	$b \to \bar{c}$, ps^{-1}	$c \to s$, ps^{-1}	PI, ps^{-1}	WA, ps^{-1}	τ_{B_c}, ps
$m_b = 5.0, m_c = 1.5, m_s = 0.20$	0.694	1.148	-0.115	0.193	0.54
$m_b = 4.8, m_c = 1.35, m_s = 0.15$	0.576	0.725	-0.132	0.168	0.75
$m_b = 5.1, m_c = 1.6, m_s = 0.45$	0.635	1.033	-0.101	0.210	0.55
$m_b = 5.1, m_c = 1.6, m_s = 0.20$	0.626	1.605	-0.101	0.210	0.43
$m_b = 5.05, m_c = 1.55, m_s = 0.20$	0.623	1.323	-0.107	0.201	0.48
$m_b = 5.0, m_c = 1.5, m_s = 0.15$	0.620	1.204	-0.114	0.193	0.53

Along with the quark masses, which, to the moment, could be generally considered as quantities with quite low uncertainties, the variation of light quark wave function plays a significant role in the calculations of inclusive decay widths for the baryons with two heavy quarks. As we have explained above, this quantity was estimated under the assumption that it is analogous to the wave functions of D mesons, i.e. we suggest that the corrections to the value obtained in the framework of potential

[21] Again, one does not take into account the data on B_c.

[22] The heavy quarkonium is quite precisely described due to lots of data on the charmonium and bottomonium.

models are similar in the mesons and baryons. In the analysis of [95, 96] the following relation for the wave function of light quark in the doubly heavy baryon was supposed:

$$|\Psi_l(0)|^2 = \frac{2}{3}\frac{f_D^2 M_D k^{-\frac{4}{9}}}{12}, \tag{4.56}$$

where $f_D = 170$ MeV, and the factor of $k^{-\frac{4}{9}}$ is caused by the so-called "hybrid" logarithms for the nonrelativistic heavy quarks. Such the expression is derived if we suppose the scaling of hyperfine spin-spin splitting in the charmed mesons and baryons and take into account corresponding spin factors as well as the double mass of diquark composed by two heavy quarks. The assumption on the independence of splitting on the mesonic or baryonic state looks quite speculative, while, nevertheless, if we leave physical motivations, the numerical effect is the reduction of wave function of light quark by two or three times. In contrast, the calculations in the framework of potential models result in the twice enhancement of wave function factor. Thus, the numerical value accepted in previous sections gives the central value for the widths under the variation of wave function of light quark.

Table 4.6. The lifetime and inclusive widths of Ξ_{cc}^{++} baryon.

Parameters, GeV	$\sum c \to s$, ps^{-1}	PI, ps^{-1}	$\tau_{\Xi_{cc}^{++}}$, ps
$m_c = 1.35, m_s = 0.15$	1.638	-0.616	0.99
$m_c = 1.6, m_s = 0.45$	2.397	-0.560	0.56
$m_c = 1.55, m_s = 0.2$	3.104	-0.874	0.45

Table 4.7. The lifetime and inclusive widths of Ξ_{cc}^{+} baryon.

Parameters, GeV	$\sum c \to s$, ps^{-1}	WS, ps^{-1}	$\tau_{\Xi_{cc}^{+}}$, ps
$m_c = 1.35, m_s = 0.15$	1.638	1.297	0.34
$m_c = 1.6, m_s = 0.45$	2.397	2.563	0.20
$m_c = 1.55, m_s = 0.2$	3.104	1.776	0.20

Table 4.8. The lifetime and inclusive widths of Ω_{cc}^{+} baryon.

Parameters, GeV	$\sum c \to s$, ps^{-1}	PI, ps^{-1}	$\tau_{\Omega_{cc}}$, ps
$m_c = 1.35, m_s = 0.15$	1.638	1.780	0.30
$m_c = 1.6, m_s = 0.45$	2.397	0.506	0.34
$m_c = 1.55, m_s = 0.2$	3.104	1.077	0.24

For the sake of presentation on the degree of variations in the theoretical predictions for the inclusive widths of doubly heavy baryons we show the estimates from [96] exploring the underestimated value of wave function of light quark, in Tables 4.6-4.8, which should be compared with the results in Table 4.1.

Remember that the estimates with the low value of charmed quark mass give the illustrations, only, and they cannot be correct because of the contradiction with the data on the lifetime of B_c.

Summimg up the analyzed uncertainties caused by the masses and wave function of light quark in the baryon, we present our final estimates in Table 4.9.

Table 4.9. The lifetimes of doubly heavy baryons.

baryon	τ, ps	baryon	τ, ps	baryon	τ, ps
Ξ_{cc}^{++}	0.46 ± 0.05	Ξ_{bc}^{+}	0.30 ± 0.04	Ξ_{bb}^{0}	0.79 ± 0.02
Ξ_{cc}^{+}	0.16 ± 0.05	Ξ_{bc}^{0}	0.27 ± 0.03	Ξ_{bb}^{-}	0.80 ± 0.02
Ω_{cc}^{+}	0.27 ± 0.06	Ω_{bc}^{0}	0.22 ± 0.04	Ω_{bb}^{-}	0.80 ± 0.02

In ref. [97] the analysis comparing the structures of OPE for the heavy hadrons was done on the basis of symmetry in the hadronic matrix elements determining the contributions by the Pauli interference and weak scattering of constituents[23], so that up to both corrections in the inverse powers of heavy quark mass and logarithmic terms, which are given by anomalous dimensions of corresponding operators, the following scaling relations were derived:

$$\frac{\Gamma[B^-]-\Gamma[B^0]}{\Gamma[D^+]-\Gamma[D^0]} = \frac{\Gamma[\Xi_b^-]-\Gamma[\Xi_b^0]}{\Gamma[\Xi_c^+]-\Gamma[\Xi_c^0]} = \frac{\Gamma[\Xi_{bb}^-]-\Gamma[\Xi_{bb}^0]}{\Gamma[\Xi_{cc}^{++}]-\Gamma[\Xi_{cc}^+]} = \frac{m_b^2}{m_c^2}\frac{|V_{cb}|^2}{|V_{cs}|^2}. \tag{4.57}$$

The accuracy of such relations should be of the order of 50%, since, for instance, according to the consideration of leptonic constants for the heavy mesons, the hadronic matrix elements of quark currents with the charmed and light quarks acquire large corrections about 50-90% certainly due to both the $1/m_Q$ terms and the logarithmic renormalization. Since the cc-diquark is twice heavier than the charmed quark, in the leading order we can put that the mentioned corrections could be twice smaller for the doubly heavy baryons. We have explored the data of Table 4.9 in order to test the last equality in (4.57). First of all, as we see, the accuracy of theoretical predictions does not allow us to make some convincing quantitative conclusions on the difference of lifetimes for the baryons with two b quarks. If we restrict ourselves by the difference of central values, then the studied part of relation (4.57) is really satisfied with the accuarcy of 50%, which points to a qualitative consistency of relation, whose quantitative accuracy is sadly low.

4.4. Exclusive decays in sum rules of NRQCD

In this section we describe the calculations of exclusive semileptonic cascade decays of doubly heavy baryons as well as two-paricle hadronic decays in the approximation of factorization for the weak trasition current of quarks [98].

In the framework of NRQCD sum rules, the following form of baryonic current was chosen in [98]:

$$J_{\Xi_{QQ}^{\diamond}} = \varepsilon^{\alpha\beta\gamma} : (Q_\alpha^T C\gamma_5 q_\beta) Q_\gamma :, \tag{4.58}$$

which leads to the necessary anti-symmetrization in corresponding diagrams, since two identical heavy quarks can enter the baryon. In this way, while taking the matrix element of chosen current

[23] In this way, we deal with the wave function of light quark independent on the flavor of infinitely heavy quark being the source of gluon field determining the motion of light quark.

over the physical baryonic states and the vacuum there is a component giving zero unphysical contribution, which is not essential. In this approach, the baryonic coupling constants are generally different from the values calculated above. Therefore, we have to analyze the two-point correlation functions with the new choice of current structure, that was done in [98]. However, the new choice has a formal advantage in the consideration of three-point correlators determining, for instance, the formfactors of semileptonic decays (see Fig. 4.4).

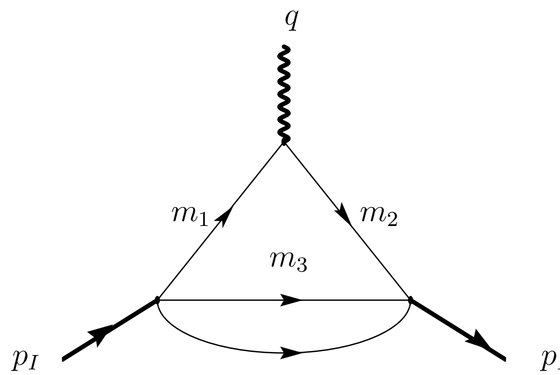

Figure 4.4. The quark loop for the thre-point correlator in the decay of baryon.

Consider the correlator

$$\Pi_\mu = i^2 \int d^4x d^4y \langle 0|T\{J_{H_I}(y) J_\mu(0) \bar{J}_{H_F}(x)\}|0\rangle \, e^{ip_F\cdot x} e^{-ip_I\cdot y}, \tag{4.59}$$

where J_μ is the weak current of quark decay. The theoretical part of sum rules can be expressed in the form of dispersion relation

$$\Pi_\mu^{(theor)}(s_1, s_2, q^2) = \frac{1}{(2\pi)^2} \int_{m_I^2}^{\infty} ds_1 \int_{m_F^2}^{\infty} ds_2 \frac{\rho_\mu(s_1, s_2, q^2)}{(s_1 - s_1^0)(s_2 - s_2^0)} + \dots \tag{4.60}$$

where dots denote possible subtractions providing the convergency of integrals, and the spectral densities were calculated in [98] in the limit of spin symmetry in the effective lagrangian of heavy nonrelativistic quarks under account of quark loop and corrections due to the condensate of light quarks. In the approximation of symmetry the only scalar correlator should be calculated. Indeed, the hadronic part od sum rules has the form

$$\Pi_\mu^{(phen)}(s_1, s_2, q^2) = \sum_{spins} \frac{\langle 0|J_{H_F}|H_F(p_F)\rangle}{s_2^0 - M_{H_F}^2} \, \langle H_F(p_F)|J_\mu|H_I(p_I)\rangle \, \frac{\langle H_I(p_I)|\bar{J}_{H_I}|0\rangle}{s_1^0 - M_{H_I}^2}, \tag{4.61}$$

where the formfactor of decay for the baryon with the spin $\frac{1}{2}$ into the baryon with the spin $\frac{1}{2}$ is expressed in the general form

$$\begin{aligned} \langle H_F(p_F)|J_\mu|H_I(p_I)\rangle = \bar{u}(p_F) \quad & \{\gamma_\mu G_1^V + v_\mu^I G_2^V + v_\mu^F G_3^V + \\ & \gamma_5(\gamma_\mu G_1^A + v_\mu^I G_2^A + v_\mu^F G_3^A)\} \; u(p_I). \end{aligned} \tag{4.62}$$

In general, all of six formfactors are independent. However, in the leading order the lagrangian of NQRCD possesses the symmetry, so that at small recoil momenta restricting the virtualities of gluonic exchanges in the hadronic state we can derive relations connecting the formfactors with each other, if they give nonzero contributions. So, in this limit the 4-velocities of baryons in the initial and final states slightly deviate from each other $v_I \neq v_F$, while their scalar product is close to unit $w = (v_I \cdot v_F) \to 1$. Then, the correlation function for the decay of heavy quark into the heavy quark has the form

$$\Pi_\mu^{(theor)} \sim \xi^{IW}(w)(1 + \not{\tilde{v}}_F)\gamma_\mu(1 - \gamma_5)(1 + \not{\tilde{v}}_I), \tag{4.63}$$

where

$$\tilde{v}_I = v_I + \frac{m_3}{2m_1}(v_I - v_F), \tag{4.64}$$

$$\tilde{v}_F = v_F + \frac{m_3}{2m_2}(v_F - v_I). \tag{4.65}$$

We see that the correlation function is determined by the only formfactor ξ^{IW} at the minimal recoil momentum. However, this formfactor is not universal, i.e. it depends on the quark contents of baryons in the process of decay.

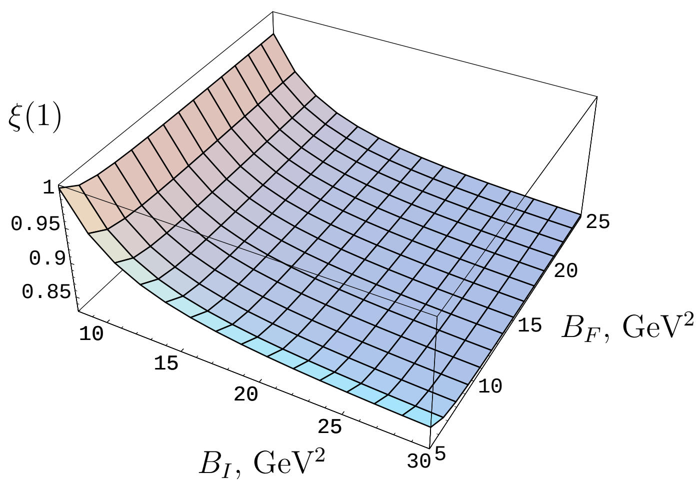

Figure 4.5. The forfactor $\xi(1)$ for the transition of $\Xi_{bb}^{\diamond} \to \Xi_{bc}^{\diamond}$ in the Borel scheme of sum rules.

For the decay of heavy quark into the light one we find

$$\Pi_\mu^{(theor)} \sim \{\xi_1(w)\not{v}_I + \xi_2(w)\not{v}_F + \xi_3(w)\}\gamma_\mu(1 - \gamma_5)(1 + \not{\tilde{v}}_I), \tag{4.66}$$

wherefrom we can get the spin-symmtry relation

$$(G_1^V + G_2^V + G_3^V) = \xi^{IW}(w), \tag{4.67}$$

$$G_1^A = \xi^{IW}(w) \tag{4.68}$$

and the following connection between the functions $\xi_i(w)$ takes place:

$$\xi_1(w) + \xi_2(w) = \xi_3(w) = \xi^{IW}(w).$$

Exploring the presence of heavy hadrons in both the initial and final states, we draw the conclusion that two form factors $G_1^V = G_1^A = \xi(w)$ are not suppressed by the heavy quark mass, only.

The formfactor ξ at nozero recoils is determined in accordance with

$$\begin{aligned}\xi^{IW}(w) &= \frac{1}{(2\pi)^2}\frac{1}{8M_I M_F Z_I Z_F}\int_{(m_1+m_3)^2}^{s_I^{th}}\int_{(m_1+m_2)^2}^{s_F^{th}} \rho(s_I, s_F, q^2) ds_I ds_F \\ &\quad \times \exp(-\frac{s_I - M_I^2}{B_I^2})\exp(-\frac{s_F - M_F^2}{B_F^2}),\end{aligned} \tag{4.69}$$

where B_I and B_F are the parameters of Borel transform over the invariant squares of masses in the initial and final states of decay, while Z_I and Z_F are the coupling constants of baryons with the appropriate currents of quarks.

4.4.1. Numerical estimates

The estimates obtained in the sum rules for the function analogous to the Isgur–Wise function are presented in Table 4.10 for the formfactors of semileptonic decays of doubly heavy baryons into the baryonic states with the spin $\frac{1}{2}$, in comparison with the values calculated in the potential model. The deviation between the values of $\xi(1)$ in these two approaches does not lead to any discrepancy, since the corresponding systematic uncertainty is about 10%.

Table 4.10. The normalization of Isgur–Wise formfactor at zero recoil in the sum rules and potential model.

Mode	$\xi(1)$, sum rules	$\xi(1)$, pot.model
$\Xi_{bb} \to \Xi_{bc}$	0.85	0.91
$\Xi_{bc} \to \Xi_{cc}$	0.91	0.99
$\Xi_{bc} \to \Xi_{bs}$	0.9	0.99
$\Xi_{cc} \to \Xi_{cs}$	0.99	1.

The result on the normaliztion of Isgur–Wise function in the transition of $\Xi_{bb} \to \Xi_{bc}$ is presented in Fig. 4.5 obtained in the Borel scheme of NRQCD sum rules.

In order to calculate the exclusive widths we suggest that the dependence of formfactors on the transfer momentum has the form of pole

$$\xi^{IW}(w) = \xi_0 \frac{1}{1 - \frac{q^2}{m_{pole}^2}}, \tag{4.70}$$

with

$$\begin{aligned} m_{pole} &= 6.3 \quad \text{GeV for } \; b \to c \; \text{ transitions,} \\ m_{pole} &= 1.85 \;\; \text{GeV for } \; c \to s \; \text{ transitions.}\end{aligned}$$

The results of calculation for the exclusive decay widths of doubly heavy baryons in the framework of NRQCD sum rules are given in Table 4.11, where the total widths have been supposed equal to the mean values presented in the previous section. The contribution of cascade decays into the

Table 4.11. The branching ratios (Br) for the exclusive decays of baryons with two heavy quarks.

Mode	Br (%)	Mode	Br (%)
$\Xi_{bb}^{\diamond} \to \Xi_{bc}^{\diamond} l\bar{\nu}_l$	14.9	$\Xi_{bc}^{+} \to \Xi_{cc}^{++} l\bar{\nu}_l$	4.9
$\Xi_{bc}^{0} \to \Xi_{cc}^{+} l\bar{\nu}_l$	4.6	$\Xi_{bc}^{+} \to \Xi_{b}^{0} l\nu_l$	4.4
$\Xi_{bc}^{0} \to \Xi_{b}^{-} l\nu_l$	4.1	$\Xi_{cc}^{++} \to \Xi_{c}^{+} l\nu_l$	16.8
$\Xi_{cc}^{+} \to \Xi_{c}^{0} l\nu_l$	7.5	$\Xi_{bb}^{\diamond} \to \Xi_{bc}^{\diamond} \pi^-$	2.2
$\Xi_{bb}^{\diamond} \to \Xi_{bc}^{\diamond} \rho^-$	5.7	$\Xi_{bc}^{+} \to \Xi_{cc}^{++} \pi^-$	0.7
$\Xi_{bc}^{0} \to \Xi_{cc}^{+} \pi^-$	0.7	$\Xi_{bc}^{+} \to \Xi_{cc}^{++} \rho^-$	1.9
$\Xi_{bc}^{0} \to \Xi_{cc}^{+} \rho^-$	1.7	$\Xi_{bc}^{+} \to \Xi_{b}^{0} \pi^+$	7.7
$\Xi_{bc}^{0} \to \Xi_{b}^{-} \pi^+$	7.1	$\Xi_{bc}^{+} \to \Xi_{b}^{0} \rho^+$	21.7
$\Xi_{bc}^{0} \to \Xi_{b}^{-} \rho^+$	20.1	$\Xi_{cc}^{++} \to \Xi_{c}^{+} \pi^+$	15.7
$\Xi_{cc}^{+} \to \Xi_{c}^{0} \pi^+$	11.2	$\Xi_{cc}^{++} \to \Xi_{c}^{+} \rho^+$	46.8
$\Xi_{cc}^{+} \to \Xi_{c}^{0} \rho^+$	33.6		

baryons with the spin $\frac{3}{2}$ is also taken into account in the table. In this estimate the results of [99] on decays $\Xi_{bc} \to \Xi_{cc} + l\bar{\nu}$ were used. In the decays with $\Xi_{bb}^{\diamond}$ and $\Xi_{cc}^{\diamond}$ the factor caused by the anti-symmetrization of identical quarks was taken into account. For the decays of $\Xi_{cc}^{++} \to \Xi_{c}^{+} X$ the correction factor of 0.62 was introduced because of negative interference of Pauli. The author of [98] claims that the values obtained in these sum rules agree with the results in the potential approach of [99] as well as with the above estimates of inclusive decay widths if we sum up the corresponding exclusive widths calculated in the sum rules.

4.5. Discussion

In this chapter we have calculated the lifetimes of baryons with two heavy quarks in the framework of consistent consideration of OPE in the inverse powers of heavy quark masses. In this expansion the leading constribution is determined by the spectator widths of heavy quarks in the inclusive decays, while the significant corrections appear under the taking into account the effects of Pauli interference and weak scattering, which contribute about 20-30% for the baryons Ξ_{cc} and Ξ_{bc}. The measurement of lifetimes for the doubly heavy baryons allows us to make the comparative analysis of deacy mechanisms for the hadrons with heavy quarks, that is especially actual in the light of searching for the fine effects with the violation of combined -parity in the sector of heavy quarks, because the characteristics of quark inteactions enter the measured quantities with the factors composed by the hadronic matrix elements of quark currents. A reliable knowledge of properties for such matrix elements could be essentially enriched by the study of decays and lifetimes of baryons with two heavy quarks. Such the investigations allow us to numerically analyze possible effects with the violation of quark-hadron duality (these effects should be small, as we have stressed). An actual challenge is studying the dependence of heavy quark decay widths on the contents of hadron, which could be essential in the clarification of reasons causing the large deviation from unit for the ration of Λ_b and B lifetimes. The measurement of doubly heavy baryon lifetimes allows us also to investigate the characteristics of confinement for the heavy quarks in various systems.

An open field of activity is a calculation of exclusive decay widths for the baryons with two heavy quarks. The presented estimates in the framework of NQRCD sum rules and potential models are

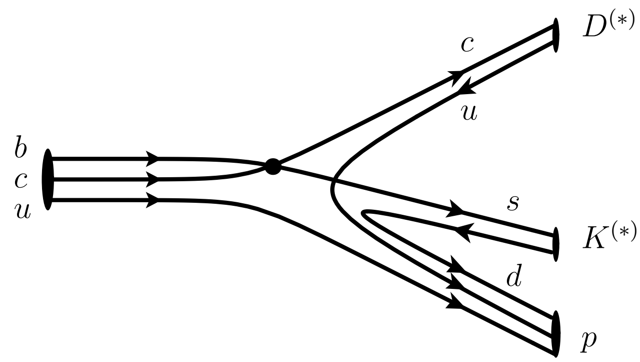

Figure 4.6. The decays of $\Xi_{bc} \to D^{(*)}K^{(*)}p$ in the process with the weak scattering of constituent b and c quarks.

quite preliminary, of course, since the challenge here is a role of corrections in the inverse powers of heavy quark mass, which could be very significant for the hadrons containing the charmed quark.

Moreover, the studies of exclusive hadronic decays for the doubly heavy baryons are of interst for the experimental practice. Some channels have to be distiguished among the hadronic decays. In contrast to the cascade decays of two heavy quarks, for example, $\Xi_{bc} \to \Xi_{cc}+X \to \Xi_c+X \to \Xi+X$, which require a reconstruction of three "secondary" vertices, there are the processes with the weak scattering of constituents, in part, the heavy quarks, that will lead to the only heavy quark in the final state as it happens in Fig. 4.6 for Ξ_{bc}, so that the only heavy hadron should be detected. The contribution of weak scattering into the total width is large enough (about 20%). Simple estimates of suppression factors show that the branching fraction of such decay is about several tenth per cent.

We believe that the experimental investigations of doubly heavy baryons are quite actual problem, first of all, at the hadron colliders, and the measurements of their decay characteristics could significantly enrich the knowledges on the mechanisms of heavy quark decays.

Conclusion

In this review we have presented the basic physical characteristics of baryons containing two heavy quarks. The description of such hadrons is based on the hierarchy of scales in the interactions determining the time intervals and distances ascribed for the strong interactions in the subsystems composing these baryons. So, because of the nonrelativistic motion of heavy quarks with respect to each other the time interval for forming the system of two heavy quarks is greater than the time period for the hard production of heavy quarks or “dressing” the quarks by hard gluons. On the other hand, this time for the forming the heavy diquark is less than the characteristic time interval for the interactions providing the confinement of light quarks involving the low-frequency strong-interacting fields. Due to the mentioned hierarchy of strong interactions in the quark systems under consideration we have developed the methods of heavy quark effective theory for the baryons with two heavy quarks. In the effective theory we have isolated the leading approximation and constructed the systematic way for the calculation of corrections to it.

Having started from the general approaches in the description of hadronic systems with the heavy quarks, we have developed the following particular methods for the consideration of baryons with two heavy quarks:

- the potential of static quarks and the potential model for the doubly heavy baryons, the separation between the heavy quark motion inside the diquark and the light quark motion in the field of diquark;

- the formulation of two-point sum rules in NRQCD for the quark currents corresponding to the baryons with two heavy quarks, and the calculations of both ground state masses for such the baryons and their coupling constants with the currents, taking into account the corrections to the local condensates of light and strange quarks;

- the calculation of anomalous dimensions for the baryonic currents with two heavy quarks in NRQCD;

- the scaling functions of diquark fragmentation;

- the numerical calculations for the complete set of diagrams in the fourth order of QCD and the analysis of higher twists over the transverse momentum;

- the generalization of OPE for the inclusive widths of baryons with two heavy quarks and a single light quark;

- the formulation of three-point sum rules in NQRCD for the exclusive semileptonic decays and hadronic decays in the approximation of factorization for the transition current.

The most bright physical effects in the baryons containing two heavy quarks are the following:

- there are the families of doubly heavy baryons, i.e. the systems of quasi-stable excited levels for the baryons with identical heavy quarks due to a suppression of operators determining the transitions into the low-lying and ground states, since the quantum numbers of diquark have to change in such transitions, so that for some states the operators, which are not suppressed by the heavy quark mass or small size of diquark, have to be equal to zero because of the Pauli principle;

- the cascade processes of fragmentation at high transverse momenta, wherein we can get analytical results for the universal fragmentation functions in the perturbative QCD for the heavy quark into the heavy diquark and for the heavy diquark into the doubly heavy baryon, while the evolution of fragmentation function because of emission of hard gluons can be described in the framework of renornalization group in QCD;

- the separation of regimes for the fragmentation and recombination in hadronic processes by taking into account the higher twists over the transverse momentum, which can be described in the framework of peturbative QCD by calculating the complete set of diagrams in the given order of coupling constant;

- large, about 20-50%, contributions of nonspectator terms into the lifetimes of doubly heavy baryons depend on the valence quark contents of baryons and are given by the effects of Pauli interference and weak scattering, especially in the presense of charmed quark, that leads to a strong splitting of lifetimes:

$$\begin{array}{lclcl} \tau[\Xi_{cc}^{++}] & > & \tau[\Omega_{cc}^{+}] & > & \tau[\Xi_{cc}^{+}], \\ \tau[\Xi_{bc}^{+}] & > & \tau[\Xi_{bc}^{0}] & > & \tau[\Omega_{bc}^{0}], \\ \tau[\Xi_{bb}^{-}] & \approx & \tau[\Omega_{bb}^{-}] & > & \tau[\Xi_{bb}^{0}], \end{array}$$

- cascade mechanisms of decays for the baryons with two heavy quarks, while there are peculiar modes due to the weak scattering, which have sizable branching ratios.

Sure, direct measuring the masses of ground states and excited levels allows us to essentially move forward more deep understanding the dynamics of forming the bound states with the heavy quarks. The theory for the radiative transitions between the quasi-stable levels of doubly heavy baryons due to either the electromagnetic or strong forces should be developed, wherein the method of chiral lagrangian should be generalized for the soft emission of Goldstone's mesons, particularly, pions.

A quite complete picture of production mechanisms has been constructed for the doubly heavy baryons, so that the search for these baryons at hadron colliders with high luminosities could be prospectively proposed. The distributions over the transverse momentum of baryons could give the most important information on the regimes of production and probably make a contribution into the understanding of reasons resposible for the discrepance between the theory and the experimental data on the yield of b-hadrons in hadron collisions.

The experimental data on the lifetimes of doubly heavy baryons could represent the most interesting information, since they are strictly connected to the whole system for describing the inclusive decays of heavy hadrons. The branching ratios of semileptonic decays could be important for determining the role of gluon corrections to the nonleptonic lagrangian for the weak charged currents of quarks. The lifetimes would allow us to enrich the knowledge on the masses of heavy quarks and the relative contributions of various decay mechanisms, since the total widths are extremely sensitive to the above physical characteristics.

We could suppose that the description of exclusive hadronic decays would require extensive theoretical efforts for the baryons with two heavy quarks.

Thus, the physics of baryons containing two heavy quarks is sufficiently rich and informative, so that it will justly occupy a honour place in the experimental investigations, in the light of optimistic occurancies and tendancies depicted by the modern theory, which deals with this field entering the time of mature progress.

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Appendix I. The coefficients of spectral densities

The spectral densities in (2.19) have the coefficients

$$\begin{aligned}
\eta_{1,0}(\omega) &= 16\omega^2(429\mathcal{M}_{diq}^3 + 715\mathcal{M}_{diq}^2\omega + 403\mathcal{M}_{diq}\omega^2 + 77\omega^3), \\
\eta_{1,1}(\omega) &= 104\omega(231\mathcal{M}_{diq}^3 + 297\mathcal{M}_{diq}^2\omega + 121\mathcal{M}_{diq}\omega^2 + 15\omega^3), \\
\eta_{1,2}(\omega) &= \frac{10}{(\mathcal{M}_{diq}+\omega)^2}(3003\mathcal{M}_{diq}^5 + 9009\mathcal{M}_{diq}^4\omega + 9438\mathcal{M}_{diq}^3\omega^2 \\
&\quad +4290\mathcal{M}_{diq}^2\omega^3 + 871\mathcal{M}_{diq}\omega^4 + 77\omega^5).
\end{aligned} \tag{A.I.1}$$

The coefficients of spectral densities in (2.20) have the form

$$\begin{aligned}
\eta_{2,0} &= 42\omega(\mathcal{M}_{diq}^2 + 48\mathcal{M}_{diq}\omega + 14\omega^2), \\
\eta_{2,1} &= 3(35\mathcal{M}_{diq}^2 + 28\mathcal{M}_{diq}\omega + 5\omega^2), \\
\eta_{2,2} &= \frac{1}{(\mathcal{M}_{diq}+\omega)^2}(105\mathcal{M}_{diq}^3 + 315\mathcal{M}_{diq}^2\omega + 279\mathcal{M}_{diq}\omega^2 + 77\omega^3).
\end{aligned} \tag{A.I.2}$$

The spectral densities taking into account the coulomb corrections in (2.24) have the coefficients

$$\begin{aligned}
\eta_{1,0}^{\mathbf{C}} &= (2\mathcal{M}_{diq}+\omega)^2\omega^2, \\
\eta_{1,1}^{\mathbf{C}} &= \frac{3(2\mathcal{M}_{diq}+\omega)\omega}{(\mathcal{M}_{diq}+\omega)}(4\mathcal{M}_{diq}^3 + 6\mathcal{M}_{diq}^2\omega + 4\mathcal{M}_{diq}\omega^2 + \omega^3), \\
\eta_{1,2}^{\mathbf{C}} &= \frac{1}{(\mathcal{M}_{diq}+\omega)^2}(12\mathcal{M}_{diq}^4 + 24\mathcal{M}_{diq}^3\omega + 32\mathcal{M}_{diq}^2\omega^2 + 20\mathcal{M}_{diq}\omega^3 + 5\omega^4).
\end{aligned} \tag{A.I.3}$$

The coefficients of spectral densities taking into account the coulomb corrections in (2.25) have the form

$$\begin{aligned}
\eta_{2,0}^{\mathbf{C}} &= (2\mathcal{M}_{diq}+\omega)\omega, \\
\eta_{2,1}^{\mathbf{C}} &= \frac{2}{\mathcal{M}_{diq}+\omega}(2\mathcal{M}_{diq}^2 + 2\mathcal{M}_{diq}\omega + \omega^2), \\
\eta_{2,0}^{\mathbf{C}} &= \frac{2}{(\mathcal{M}_{diq}+\omega)^2}(2\mathcal{M}_{diq}^2 + 2\mathcal{M}_{diq}\omega + \omega^2).
\end{aligned} \tag{A.I.4}$$

For the spectral densities with the gluon condensate in (2.30) we find

$$\begin{aligned}
\eta_{1,0}^{G^2} &= 84\mathcal{M}_{diq}^3 + 196\mathcal{M}_{diq}^2\omega + 133\mathcal{M}_{diq}\omega^2 + 11\omega^3, \\
\eta_{1,1}^{G^2} &= -\frac{2(210\mathcal{M}_{diq}^3 + 70\mathcal{M}_{diq}^2\omega + 21\mathcal{M}_{diq}\omega^2 + 3\omega^3)}{\mathcal{M}_{diq}+\omega}, \\
\eta_{1,2}^{G^2} &= \frac{2(210\mathcal{M}_{diq}^3 + 70\mathcal{M}_{diq}^2\omega + 21\mathcal{M}_{diq}\omega^2 + 3\omega^3)}{(\mathcal{M}_{diq}+\omega)^2}.
\end{aligned} \tag{A.I.5}$$

Appendix II. The distribution over the transverse momentum

For the fragmentation of vector diquark into the baryon state in the scaling limit we get the following distribution over $t = p_T/M$ with respect to the fragmentation axis:

$$D(t) = \frac{64\alpha_s^2}{81\pi}\,\frac{|R(0)|^2}{3(1-r)^5M^3}\,\frac{1}{t^6}$$

$$
\Big\{t(30r^3-30r^4-61t^2r+45r^2t^2+33r^3t^2-
-17r^4t^2+3t^4-9rt^4+15r^2t^4-9r^3t^4)-
(30r^4-99r^2t^2-54r^3t^2+27r^4t^2+9t^4+18rt^4-6r^2t^4+
+18r^3t^4+3r^4t^4+3t^6-6rt^6+9r^2t^6)\mathrm{arctg}\Big(\frac{(1-r)t}{r+t^2}\Big)+
24(2r^3t+rt^3+r^2t^3)\ \ln\Big(\frac{r^2(1+t^2)}{r^2+t^2}\Big)\Big\}. \tag{A.II.1}
$$

Appendix III. The spectator effects in the baryon decays

For the effects of Pauli interference and electroweak scattering in the baryons Ξ_{cc} we have the following expresions:

$$
\begin{aligned}
\mathcal{T}_{PI} &= -\frac{2G_F^2}{4\pi}m_c^2(1-\frac{m_c}{m_u})^2 \\
&([(\frac{(1-z_-)^2}{2}-\frac{(1-z_-)^3}{4})(\bar{c}_i\gamma_\alpha(1-\gamma_5)c_i)(\bar{q}_j\gamma^\alpha(1-\gamma_5)q_j)+ \\
&(\frac{(1-z_-)^2}{2}-\frac{(1-z_-)^3}{3})(\bar{c}_i\gamma_\alpha\gamma_5c_i)(\bar{q}_j\gamma^\alpha(1-\gamma_5)q_j)][(C_++C_-)^2+ \\
&\frac{1}{3}(1-k^{\frac{1}{2}})(5C_+^2+C_-^2-6C_-C_+)]+ \\
&[(\frac{(1-z_-)^2}{2}-\frac{(1-z_-)^3}{4})(\bar{c}_i\gamma_\alpha(1-\gamma_5)c_j)(\bar{q}_j\gamma^\alpha(1-\gamma_5)q_i)+ \\
&(\frac{(1-z_-)^2}{2}-\frac{(1-z_-)^3}{3})(\bar{c}_i\gamma_\alpha\gamma_5c_j)(\bar{q}_j\gamma^\alpha(1-\gamma_5)q_i)]k^{\frac{1}{2}}(5C_+^2+C_-^2-6C_-C_+)),
\end{aligned} \tag{A.III.1}
$$

$$
\begin{aligned}
\mathcal{T}_{WS} &= \frac{2G_F^2}{4\pi}p_+^2(1-z_+)^2[(C_+^2+C_-^2+ \\
&\frac{1}{3}(1-k^{\frac{1}{2}})(C_+^2-C_-^2))(\bar{c}_i\gamma_\alpha(1-\gamma_5)c_i)(\bar{q}_j\gamma^\alpha(1-\gamma_5)q_j)+ \\
&k^{\frac{1}{2}}(C_+^2-C_-^2)(\bar{c}_i\gamma_\alpha(1-\gamma_5)c_j)(\bar{q}_j\gamma^\alpha(1-\gamma_5)q_i)],
\end{aligned} \tag{A.III.2}
$$

where $p_+=p_c+p_q$, $p_-=p_c-p_q$ $z_\pm=\frac{m_c^2}{p_\pm^2}$. Further, for p_+ and p_- we use their threshold values

$$
p_+=p_c(1+\frac{m_q}{m_c}),\quad p_-=p_c(1-\frac{m_q}{m_c}).
$$

These expressions were derived with account of low-energy renormalization of nonleptonic lagrangian for the weak interaction of nonrealtivistic heavy quarks. This lagrangian [6, 89] has the form

$$
\begin{aligned}
L_{eff,\,log} &= \frac{G_F^2m_c^2}{2\pi}\{\frac{1}{2}[C_+^2+C_-^2+\frac{1}{3}(1-k^{\frac{1}{2}})(C_+^2-C_-^2)](\bar{c}\Gamma_\mu c)(\bar{d}\Gamma^\mu d)+ \\
&\frac{1}{2}(C_+^2-C_-^2)k^{\frac{1}{2}}(\bar{c}\Gamma_\mu d)(\bar{d}\Gamma^\mu c)+\frac{1}{3}(C_+^2-C_-^2)k^{\frac{1}{2}}(k^{-\frac{2}{9}}-1)(\bar{c}\Gamma_\mu t^a c)j_\mu^a- \\
&\frac{1}{8}[(C_++C_-)^2+\frac{1}{3}(1-k^{\frac{1}{2}})(5C_+^2+C_-^2-6C_+C_-)](\bar{c}\Gamma_\mu c+\frac{2}{3}\bar{c}\gamma_\mu\gamma_5c)(\bar{u}\Gamma^\mu u)- \\
&\frac{1}{8}k^{\frac{1}{2}}(5C_+^2+C_-^2-6C_+C_-)(\bar{c}_i\Gamma_\mu c_k+\frac{2}{3}\bar{c}_i\gamma_\mu\gamma_5c_k)(\bar{u}_k\Gamma^\mu u_i)-
\end{aligned} \tag{A.III.3}
$$

$$
\begin{aligned}
&\frac{1}{8}[(C_+ - C_-)^2 + \frac{1}{3}(1-k^{\frac{1}{2}})(5C_+^2 + C_-^2 + 6C_+C_-)](\bar{c}\Gamma_\mu c + \frac{2}{3}\bar{c}\gamma_\mu\gamma_5 c)(\bar{s}\Gamma^\mu s) - \\
&\frac{1}{8}k^{\frac{1}{2}}(5C_+^2 + C_-^2 + 6C_+C_-)(\bar{c}_i\Gamma_\mu c_k + \frac{2}{3}\bar{c}_i\gamma_\mu\gamma_5 c_k)(\bar{s}_k\Gamma^\mu s_i) - \\
&\frac{1}{6}k^{\frac{1}{2}}(k^{-\frac{2}{9}} - 1)(5C_+^2 + C_-^2)(\bar{c}\Gamma_\mu t^a c + \frac{2}{3}\bar{c}\gamma_\mu\gamma_5 t^a c)j^{a\mu}\},
\end{aligned}
$$

where $\Gamma_\mu = \gamma_\mu(1-\gamma_5)$, $k = (\alpha_s(\mu)/\alpha_s(m_c))$ and $j^a_\mu = \bar{u}\gamma_\mu t^a u + \bar{d}\gamma_\mu t^a d + \bar{s}\gamma_\mu t^a s$ is the color current of light quarks ($t^a = \lambda^a/2$ are the color generators).

Here we would like to make a note on the effective lagrangian terms containing the colored current of light quarks. we have neglected those terms, since they enter into the lagrangian with the factor of $k^{-\frac{2}{9}} - 1$ numerically about 0.054.

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